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Distributed Dynamic Encirclement Control for First-Order Multi-Agent Systems with Communication Delay

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Abstract— **This paper presents a novel distributed encirclement control for first-order multi-agent systems that specifically considers the issue of communication delay. Unlike previous studies, this research analyzes its impact. One key aspect of our approach is the utilization of two distributed estimators affected by communication delay to accurately estimate the location of the geometric center of targets and to estimate the maximum distance of targets to the estimated geometric center of targets. The relative desired position of agents is achievable by employing the estimated maximum distance of targets to the estimated geometric center of targets. The employment of the estimated profile of targets and the relative desired position of agents allows for improved tracking and coordination among the agents, enhancing the effectiveness of the engaged encirclement control. To assess the stability of the closed-loop system, we employ the Lyapunov technique for analysis. Finally, we conduct simulations to evaluate the effectiveness of our proposed methodology.**

Index Terms— **Multi-Agent Systems, Encirclement control, Delay, Stability Analysis.**

I. INTRODUCTION

A multi-agent system (MAS) refers to a collection of distinct agents capable of pursuing specific goals through local interactions. The concept of MAS finds its roots in various biological and social phenomena, such as birds migrating in formation, ants cooperating in colonies, and fish arranging themselves to avoid harm [1].

The consensus control remains a fundamental approach, where agents strive to reach an agreement on a common value [2]–[4]. Following consensus, the formation control has gained significant attention, focusing on agents maintaining a specific shape or arrangement while performing their assigned tasks [5]–[8].

Encirclement involves a group of agents working together to surround a target entity, be it a moving or stationary object. Successful encirclement necessitates effective cooperation and coordination among the agents [9]. Encirclement controls for MAS are typically designed to ensure that the target object remains surrounded by the agents at all times [10]. One such control is the leader approach, where a leader guides the swarm of agents around the target, and each agent adjusts its position relative to the leader [11].

The design and implementation of encirclement control for MAS pose several challenges. First, the control must be able to handle dynamic environments where the target object is moving [12]. Second, the control must be able to handle communication constraints among agents [13], such as limited range communication or communication delays. Third, the control must be distributed, meaning that each agent should be able to execute the control autonomously, without central coordination [14].

The encirclement control is categorized into two types: static encirclement [9] and dynamic encirclement [12]. Dynamic encirclement is based on a principle: a predetermined number of actual targets exist, and the agents' goal is to track and surround these targets. Once the agents have successfully encircled the targets, the next step is to initiate a rotational movement around the targets. In order to establish the dynamic encirclement control, a key prerequisite is the use of two estimators by each agent, with all agents equipped with the same pair of estimators. The first estimator facilitates the agents in estimating the geometric center of the targets. Simultaneously, the second estimator enables the agents to approximate the maximum distance between the farthest target and the estimated geometric center of targets. The idea of designing dynamic encirclement control will be used here.

The problem of communication delay arises when there is a lag or delay in the exchange of information among agents due to finite bandwidth or network congestion. Communication delay can significantly affect the performance of the control, leading to slower convergence or even instability [15], [16].

Previous studies on encirclement in the existing literature have overlooked the consideration of communication delay. In general, communication delay could vary during the process but in all various applications, a constant upper bound could be considered as the maximum value of communication delay. in this work, in terms of analyzing the effect of communication delay, a constant value will be considered in distributed parts of the encirclement control and the effect of that on the closed-loop system will be analyzed.

- To the author's best knowledge, it is the first time that an encirclement control in presence of communication delay has been designed for MASs.
- Compared with [12] which the dynamic encirclement has been studied without consideration of delay issue, this paper considers **communication delay** in distributed estimators among all agents. Also, the rotation method around targets is more appropriate and we do not need to concern the agents location during rotation.
- Compared with [16] which communication delay in consensus control has been studied, this paper considers the maximum constant communication delay in **dynamic encirclement control**.

The subsequent sections of this paper are structured as

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follows: Section II delves into problem formulation and dynamics. The dynamic behavior of distributed estimators is detailed in Section III. In Section IV, the encirclement control is presented. Section V investigates a simulation example, and finally, Section VI concludes the paper and outlines avenues for future research in this domain.

II. Problem Statement

In this paper, agents and targets are working in pairs. The exact position of each target is accessible by the agent with identical index and the number of targets and agents are both n. Let $\Psi = \{1, 2, ..., n\}$ be the set of indexes. All agents are in a distributed network that they are communicating with each other and their communication topology is described by an undirected simple graph $G(\Psi, E)$. Note that $G(\Psi, E)$ is connected and static during all process. Note that the position of the target i is detectable and only detected by agent $i \in \Psi$. In a connected graph, the shortest path between two nodes i and j is represented by the symbol $\text{dis}(i, j)$ and is called the distance. In this paper, $\breve{d}$ is defined as

$$\breve{d} = \max_{i,j \in \Psi, i \neq j} \{\text{dis}(i, j)\}. \tag{1}$$

A. Target Profile

The position of target $i \in \Psi$ is $\boldsymbol{y}_i(t) = [y_{i1}(t), y_{i2}(t)]^T \in \Re^2$. Regarding the dynamic of targets, the following Assumptions hold.

Assumption 1: Suppose that $\forall i \in \Psi, \quad \boldsymbol{y}_i(t)$ is continuously differentiable and the velocity of targets is bounded as

$$\|\dot{\boldsymbol{y}}_i(t)\| \leqslant \beta, \quad \forall t > 0, \tag{2}$$

where β is a positive constant.

Assumption 2: Suppose that targets will not collide with each other, that is $\forall i, j, i \neq j \in \Psi$,

$$\boldsymbol{y}_j(t) \neq \boldsymbol{y}_i(t), \forall t > 0. \tag{3}$$

The geometric center of targets is defined as

$$\boldsymbol{y}^{\text{gc}}(t) = \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{y}_k(t). \tag{4}$$

The maximum distance of targets to the geometric center of targets is defined as

$$y^{\text{md}}(t) = \max_{i \in \Psi} \{||\boldsymbol{y}_i(t) - \boldsymbol{y}^{\text{gc}}(t)||\}. \tag{5}$$

Assumption 3: Postulate that all targets' positions are within a certain circular neighborhood of $\boldsymbol{y}^{\text{gc}}(t)$ with a c radius, that is,

$$\sup_{t>0} y^{\text{md}}(t) \leqslant c, \tag{6}$$

where c is a positive constant.

B. Agents Dynamic

Consider that all the agents have first-order dynamics as

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{u}_i(t), \quad i \in \Psi, \tag{7}$$

in which $\boldsymbol{x}_i(t) = [x_{i1}(t), x_{i2}(t)]^T \in \Re^2$ indicates the position of agent i for $t \geqslant 0$. $\boldsymbol{u}_i(t) = [u_{i1}(t), u_{i2}(t)]^T \in \Re^2$ represents the input control of agent i. Note the Cartesian coordinate transformation into polar coordinate, $\boldsymbol{x} \mapsto (\rho, \theta)$ as

$$\rho_i(t) = \sqrt{x_{i1}^2(t) + x_{i2}^2(t)}, \tag{8}$$

$$\theta_i(t) = \arctan\left(\frac{x_{i2}(t)}{x_{i1}(t)}\right), \tag{9}$$

where $\rho_i(t) \in \Re$ is the polar radius, $\theta_i(t) \in \Re$ is the polar angle of agent i. Using transformation (8)-(9), the system (7), for $i \in \Psi$, will be represented in polar coordinate as

$$\dot{\rho}_i(t) = u_{i1}(t)\cos(\theta_i(t)) + u_{i2}(t)\sin(\theta_i(t)), \tag{10}$$

$$\dot{\theta}_i(t) = \rho_i^{-1}(t)(u_{i2}(t)\cos(\theta_i(t)) - u_{i1}(t)\sin(\theta_i(t))). \tag{11}$$

Definition 1: Let $\nu_i(t) \in C^1[0, \infty)$ be an arbitrarily chosen function standing for a desired encirclement rotation profile of agents. For any agent $i \in \Psi$, an input control $\boldsymbol{u}_i(t) \in C[0, \infty)$ is a **dynamic encirclement control** for the system (7) if the following equations hold:

$$\lim_{t\to\infty} \rho_i(t) = \xi_i y^{\text{md}}(t) + \|\boldsymbol{y}^{\text{gc}}(t)\|, \tag{12}$$

$$\lim_{t\to\infty} \theta_i(t) = \nu_i(t), \tag{13}$$

where $\xi_i > 1$ is a positive design parameter.

Equation (12) and (13) in Definition 1, by taking into consideration of matched initial condition, are equivalent to the following respectively

$$\lim_{t\to\infty} (\rho_i(t) - \|\boldsymbol{y}^{\text{gc}}(t)\|) = \xi_i y^{\text{md}}(t), \tag{14}$$

$$\lim_{t\to\infty} \omega_i(t) = \dot{\nu}_i(t) := \mu_i(t), \tag{15}$$

where $\omega_i(t) = \dot{\theta}_i(t) \in \Re$ is the polar velocity and $\mu_i(t)$ is the desired angular velocity of agent i which will be chosen by designer.

Assumption 4: Assume that the first derivative of desired angular velocity $\mu_i(t)$ along time is bounded that have

$$|\dot{\mu}_i(t)| \leqslant \bar{\mu}, \forall i \in \Psi, t > 0, \tag{16}$$

where $\bar{\mu} \in \Re$ is a positive constant.

Remark 1: The equation (14) guarantees that all agents will approach to a unique circular neighborhood of geometric center of targets and form the encirclement. The equation (15) guarantees that the angular velocity of agents will be equal to a time-varing function during the rotation.

III. Estimation of Targets' Position

To define the encirclement control, the utilization of two distributed estimators is necessary.

A. Geometric Center of Targets Estimator

In the initial stage of defining the encirclement control, each agent requires an estimator to estimate the geometric center of the targets. To fulfill this requirement, each agent must possess its own distributed geometric center estimator. Note that $\hat{\boldsymbol{y}}_i^{\text{gc}}(t) = [\hat{y}_{i1}^{\text{gc}}(t), \hat{y}_{i2}^{\text{gc}}(t)]^T \in \Re^2$ is the estimated geometric center of targets by agent i. The dynamic of geometric center estimator of agent i is proposed as

$$\hat{\boldsymbol{y}}_i^{\text{gc}}(t) = \boldsymbol{\delta}_i(t) + \boldsymbol{y}_i(t), \tag{17a}$$

$$\dot{\boldsymbol{\delta}}_i(t) = \begin{cases} \alpha \sum\limits_{j \in N_i} \frac{\hat{\boldsymbol{y}}_j^{\text{gc}}(t-d) - \hat{\boldsymbol{y}}_i^{\text{gc}}(t-d)}{\|\hat{\boldsymbol{y}}_j^{\text{gc}}(t-d) - \hat{\boldsymbol{y}}_i^{\text{gc}}(t-d)\|}, \\ \qquad \text{if} \quad \hat{\boldsymbol{y}}_j^{\text{gc}}(t) \neq \hat{\boldsymbol{y}}_i^{\text{gc}}(t), \quad \forall t > 0, \\ 0 \quad \text{if} \quad \hat{\boldsymbol{y}}_j^{\text{gc}}(t) = \hat{\boldsymbol{y}}_i^{\text{gc}}(t), \quad \exists t > 0, \end{cases} \tag{17b}$$

in which $\boldsymbol{\delta}_i(t) = [\delta_{i1}(t), \delta_{i2}(t)]^T$ is the intermediate state of estimator and $\boldsymbol{\delta}_i(t) = 0, \quad \forall i \in \Psi, \quad$ and $\quad \forall t \in [-d, 0)$. $\alpha > 0$ is a design parameter. d is a positive constant that represent the maximum of communication delay between agents. $\hat{\boldsymbol{y}}_i^{\text{gc}}(t) = \boldsymbol{y}_i(t), \quad \forall i \in \Psi, \quad$ and $\quad \forall t \in [-d, 0)$. a_{ij} is the relevant element of adjacency matrix of network topology.

The average of estimated geometric center of targets by agents is defined as

$$\hat{\boldsymbol{y}}^{\text{agc}}(t) := \frac{1}{n} \sum_{i=1}^{n} \hat{\boldsymbol{y}}_i^{\text{gc}}(t). \tag{18}$$

Theorem 1: Consider the geometric center estimator dynamic in equation (17), set α to 1. The estimator (17) is delay-independently asymptotically convergent to the average of estimated geometric center of targets by agents, $\hat{\boldsymbol{y}}^{\text{agc}}(t)$ as

$$\lim_{t \to \infty} \hat{\boldsymbol{y}}_i^{\text{gc}}(t) = \hat{\boldsymbol{y}}^{\text{agc}}(t), \quad i \in \Psi \tag{19}$$

The proof is omitted here because of page limitation.

Remark 2: Based on Theorem 1, the estimator dynamics (17) is asymptotically convergent. It yields that after some time, we have $\lim\limits_{t \to \infty} \dot{\boldsymbol{\delta}}_i(t) = 0$. Which also yields that $\lim\limits_{t \to \infty} \frac{1}{n} \sum\limits_{k=1}^{n} \boldsymbol{\delta}_k(t) = 0$.

Corollary 1: Based on estimated geometric center of targets definition in (17a) and the average of estimated geometric center of targets, $\hat{\boldsymbol{y}}^{\text{agc}}(t)$, we have

$$\hat{\boldsymbol{y}}^{\text{agc}}(t) = \frac{1}{n} \sum_{k=1}^{n} \hat{\boldsymbol{y}}_k^{\text{gc}}(t) = \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{\delta}_k(t) + \boldsymbol{y}^{\text{gc}}(t)$$

Taking into consideration of Theorem 1, one can conclude that the estimated geometric center of targets by agent i is convergent to the exact geometric center of targets where as

$$\lim_{t \to \infty} \hat{\boldsymbol{y}}_i^{\text{gc}}(t) = \lim_{t \to \infty} \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{\delta}_k(t) + \boldsymbol{y}^{\text{gc}}(t) = \boldsymbol{y}^{\text{gc}}(t) \quad i \in \Psi,$$

where Remark 2 has been used.

B. Maximum Distance of Targets Estimator

In the subsequent step, an estimator is required to estimate the maximum distance between the estimated geometric center to the targets. To accomplish this, each agent must possess its own maximum distance estimator. Define $\hat{y}_i^{\text{md}}(t) \in \Re$ as the estimated maximum distance between targets to their estimated geometric center by agent i. For each agent i, the estimator design for the maximum distance between targets to their estimated geometric center is

$$\dot{\hat{y}}_i^{1}(t) = -\kappa \text{sign}\left(\hat{y}_i^{1}(t-d) - \max_{j \in N_i \cup \{i\}} \{h_j(t-d)\}\right), \tag{20a}$$

$$\dot{\hat{y}}_i^{2}(t) = -\kappa \text{sign}\left(\hat{y}_i^{2}(t-d) - \max_{j \in N_i \cup \{i\}} \{\hat{y}_j^{1}(t-d)\}\right), \tag{20b}$$

$$\vdots$$

$$\dot{\hat{y}}_i^{\breve{d}-1}(t) = -\kappa \text{sign}\left(\hat{y}_i^{\breve{d}-1}(t-d) - \max_{j \in N_i \cup \{i\}} \{\hat{y}_j^{\breve{d}-2}(t-d)\}\right), \tag{20c}$$

$$\dot{\hat{y}}_i^{\text{md}}(t) = -\kappa \text{sign}\left(\hat{y}_i^{\text{md}}(t-d) - \max_{j \in N_i \cup \{i\}} \{\hat{y}_j^{\breve{d}-1}(t-d)\}\right), \tag{20d}$$

where $\breve{d}$ is from (1), and $\hat{y}_i^{\hat{j}}(t) \in \Re, \quad \forall \hat{j} \in \{1, 2, ..., \breve{d}-1\}$ are intermediate states of the estimator i. Also κ is a positive control gain to be determined; and $\hat{y}_i^{\hat{j}}(t) = 0 \quad$ for $\quad \forall t \in [-d, 0)$. $d > 0$ is the positive constant maximum of communication delay between agents network. Define

$$h_i(t) = \|\boldsymbol{y}_i(t) - \hat{\boldsymbol{y}}_i^{\text{gc}}(t)\|. \tag{21}$$

Remark 3: Based on Theorem 1, $h_i(t)$ in equation (21), we have

$$\lim_{t \to \infty} h_i(t) = \|\boldsymbol{y}_i(t) - \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{y}_k(t)\|. \tag{22}$$

The maximum distance of targets corresponding to agent i and its neighbors, to the estimated geometric center is defined as

$$h_i^{\max}(t) = \max_{j \in N_i \cup \{i\}} \{\|\boldsymbol{y}_j(t) - \frac{1}{n} \sum_{k=1}^{n} {\boldsymbol{y}^{\text{gc}}}_k(t)\|\}. \tag{23}$$

Theorem 2: Consider the geometric center estimator dynamic in equation (20), and set κ to 1. The estimator (20) is delay-independently asymptotically convergent to maximum distance of all targets to their estimated geometric center.

$$\lim_{t \to \infty} \hat{y}_i^{\text{md}}(t) = h_i^{\max}(t), \quad i \in \Psi. \tag{24}$$

The proof is omitted here because of page limitation.

IV. Encirclement Control

By utilizing the estimated data provided by the estimators, we can derive the relative position of targets, facilitating the establishment of a control to track the distance between each target and its estimated geometric center.

A. Relative Desired Position

The goal of this part is to let the relative desired position converge to $\xi_i \hat{y}_i^{\text{md}}(t)$, where we choose the coefficient $\xi_i > 1$ to guarantee collision avoidance among the agents. Because $\hat{y}_i^{\text{md}}(t)$ is a distance variable, we first design the relative desired position of the agent $\tilde{x}_i(t)$ that we want our system to track in polar coordinate and then transform it to Cartesian coordinate through (25)

$$\tilde{\boldsymbol{x}}_i(t) = [\tilde{\rho}_i(t)\cos(\tilde{\theta}_i(t)), \tilde{\rho}_i(t)\sin(\tilde{\theta}_i(t))]^T. \quad (25)$$

In terms of designing $\tilde{x}_i(t)$ out of the estimated maximum distance of targets to their geometric center $\hat{y}_i^{\text{md}}(t)$, first consider $\tilde{\rho}_i(t) \in \Re[0, \infty)$ and $\tilde{\theta}_i(t) \in \Re$ as desired relative polar radius and desired relative polar angle to be designed, respectively where we have $\tilde{\rho}_i(t)$ and $\tilde{\theta}_i(t)$ have the following dynamic

$$\dot{\tilde{\rho}}_i(t) = -\gamma_1 \text{sign}(\tilde{\rho}_i(t) - \xi_i \hat{y}_i^{\text{md}}(t)), \quad (26a)$$

$$\dot{\tilde{\theta}}_i(t) = \tilde{\omega}_i(t), \quad (26b)$$

$$\dot{\tilde{\omega}}_i(t) = -\gamma_2 \text{sign}(\tilde{\omega}_i(t) - \mu_i(t)), \quad (26c)$$

where γ_1 and γ_2 are positive design parameters and $\xi_i > 1$ is the proportional distance of agents from targets. The initial conditions will depend on the systems initial condition and will be chosen arbitrary. $\mu_i(t) \in \Re$ is desired angular velocity of each agent $\forall i \in \Psi$. $\forall i \in \Psi, \quad \xi_i$ will be defined as

$$\xi_{i+1} = \xi_i + v, \quad (27)$$

where v is positive design parameter. The choice of the parameter v in the encirclement control ensures a guaranteed distance between agents as they perform the encirclement. As part of the control, agents will rotate around the targets on circles with varying radius. This configuration allows for coordinated movement and maintains the desired spatial arrangement throughout the encirclement process.

Theorem 3: Consider the desired polar coordinate control (26) then by designing $\gamma_1 > 2\beta\xi_n$ and $\gamma_2 > \bar{\mu}$, the system (26) along together are asymptotically convergent to the desired position where $\tilde{\rho}_i(t)$ is convergent to ξ_i times bigger than estimated maximum distance of all targets to their estimated geometric center and $\tilde{\omega}_i(t)$ is convergent to desired angular velocity $\mu_i(t)$, chosen by designer. And $\forall i \in \Psi$, we have

$$\lim_{t \to \infty} \tilde{\rho}_i(t) = \xi_i \hat{y}_i^{\text{md}}(t), \quad (28)$$

$$\lim_{t \to \infty} \tilde{\omega}_i(t) = \mu_i(t). \quad (29)$$

The proof is omitted here because of page limitation.

The desired relative agent position $\tilde{\boldsymbol{x}}_i(t) \in \Re^2$ with respect to the geometric center in Cartesian coordinate is transformed by (25).

B. Main control

The following control is proposed for dynamic encirclement for the system (7) along with (17), (26), and (25),

$$\boldsymbol{u}_i(t) = -\gamma_3 \text{sign}(\boldsymbol{x}_i(t) - \hat{\boldsymbol{x}}_i(t)), \quad (30a)$$

$$\hat{\boldsymbol{x}}_i(t) = \hat{\boldsymbol{y}}_i^{\text{gc}}(t) + \tilde{\boldsymbol{x}}_i(t), \quad (30b)$$

where $\gamma_3 > 0$ is a design parameter. $\hat{\boldsymbol{x}}_i(t) \in \Re^2$ is the desired relative location of agents with respect to relative desired position and geometric center of targets. The initial conditions will be chosen arbitrary. Based on transformation (8)-(9), it is easy to map the $\boldsymbol{u}_i(t)$ to $(u_i^{\rho}(t), u_i^{\theta}(t))$ in (10)-(11) as

$$\begin{aligned} u_i^{\rho}(t) = & -\gamma_3 \{\text{sign}(\rho_i(t)\cos(\theta_i(t)) - \tilde{\rho}_i(t)\cos(\tilde{\theta}_i(t)) \\ & - \hat{y}_{i1}^{\text{gc}}(t))\cos(\theta_i(t)) + \text{sign}(\rho_i(t)\sin(\theta_i(t)) \\ & - \tilde{\rho}_i(t)\sin(\tilde{\theta}_i(t)) - \hat{y}_{i2}^{\text{gc}}(t))\sin(\theta_i(t))\}, \\ u_i^{\theta}(t) = & -\gamma_3(\rho_i(t)\cos^2(\theta_i(t))(1 + \tan^2(\theta_i(t))))^{-1} \\ & \times \{\text{sign}(\rho_i(t)\sin(\theta_i(t)) - \tilde{\rho}_i(t)\sin(\tilde{\theta}_i(t)) \\ & - \hat{y}_{i2}^{\text{gc}}(t))\cos(\theta_i(t)) - \text{sign}(\rho_i(t)\cos(\theta_i(t)) \\ & - \tilde{\rho}_i(t)\cos(\tilde{\theta}_i(t)) - \hat{y}_{i1}^{\text{gc}}(t))\sin(\theta_i(t))\}. \end{aligned}$$

Theorem 4: Consider the proposed control in (30) then by designing $\gamma_3 > \beta + 2\beta k_n + 2c\bar{\mu}k_n$, the system (7) is asymptotically convergent to desired location of agents. And we have

$$\lim_{t \to \infty} \boldsymbol{x}_i(t) = \hat{\boldsymbol{x}}_i(t), \quad i \in \Psi, \quad (31)$$

which satisfies the dynamic encirclement control.
The proof is omitted here because of page limitation.

By taking into consideration of (7) along with (17), (26), (25), and (26), the model that represents the whole closed-loop system is

$$\dot{\boldsymbol{x}}_i(t) = -\gamma_3 \text{sign}(\boldsymbol{x}_i(t) - \hat{\boldsymbol{y}}_i^{\text{gc}}(t) - \tilde{\boldsymbol{x}}_i(t)). \quad (32)$$

Fig. 1 is representing a block diagram of closed-loop system.

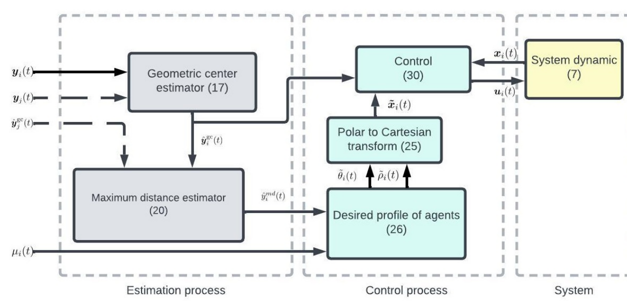

Fig. 1. Closed-Loop Block Diagram of Agent i

V. Simulation Example

In this section, we propose a numerical example to indicate the effectiveness of the theoretical results. Suppose we have 5 targets then we consider equal number of agents as 5. The communication topology among 5 agents is illustrated in Fig. 2.

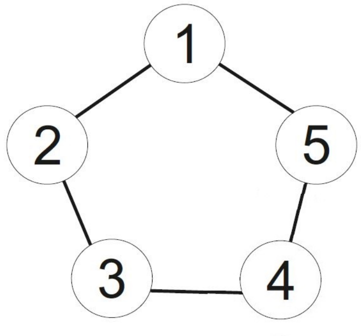

Fig. 2. Communication topology between agents

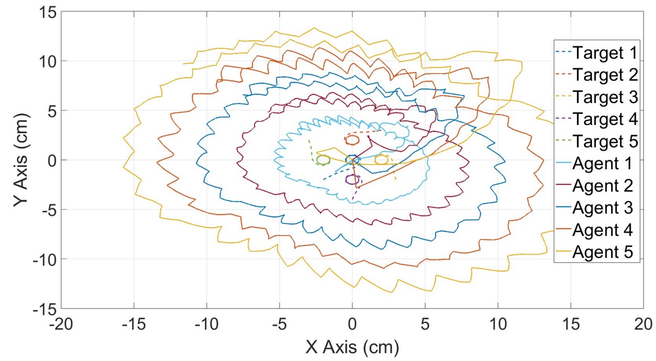

Fig. 4. The trajectories of agents' and targets' movement evolution. Communication delay is ($d = 0.1s$)

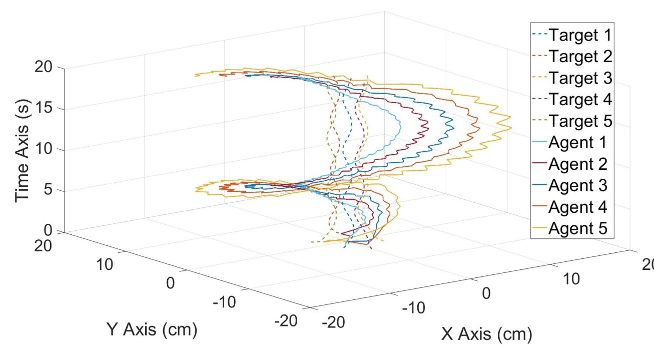

Fig. 5. The trajectories of agents' and targets' movement evolution where the vertical axis is time. Communication delay is ($d = 0.1s$)

The dynamic of targets are inspired based on limit-cycle-based design of formation control [17] as

$$\begin{cases} \dot{y}_{11}(t) = -y_{12}(t) - y_{11}(t)(y_{11}^2(t) + y_{12}^2(t) - 0.2), \\ \dot{y}_{12}(t) = y_{11}(t) - y_{12}(t)(y_{11}^2(t) + y_{12}^2(t) - 0.2), \end{cases}$$

$$\begin{cases} \dot{y}_{21}(t) = -(y_{22}(t) - 2) - y_{21}(t)((y_{22}(t) - 2)^2 + y_{21}^2(t) - 0.2), \\ \dot{y}_{22}(t) = y_{21}(t) - (y_{22}(t) - 2)((y_{22}(t) - 2)^2 + y_{21}^2(t) - 0.2), \end{cases}$$

$$\begin{cases} \dot{y}_{31}(t) = -y_{32}(t) - (y_{31}(t) - 2)(y_{32}^2(t) + (y_{31}(t) - 2)^2 - 0.2), \\ \dot{y}_{32}(t) = (y_{31}(t) - 2) - y_{32}(t)(y_{32}^2(t) + (y_{31}(t) - 2)^2 - 0.2), \end{cases}$$

$$\begin{cases} \dot{y}_{41}(t) = -(y_{42}(t) + 2) - y_{41}(t)((y_{42}(t) + 2)^2 + y_{41}^2(t) - 0.2), \\ \dot{y}_{42}(t) = y_{41}(t) - (y_{42}(t) + 2)((y_{42}(t) + 2)^2 + y_{41}^2(t) - 0.2), \end{cases}$$

$$\begin{cases} \dot{y}_{51}(t) = -y_{52}(t) - (y_{51}(t) + 2)((y_{51}(t) + 2)^2 + y_{52}^2(t) - 0.2), \\ \dot{y}_{52}(t) = (y_{51}(t) + 2) - y_{52}(t)((y_{51}(t) + 2)^2 + y_{52}^2(t) - 0.2), \end{cases}$$

where Assumptions 1, and 3 hold with $\beta = 0.5$ and $c = 0.6$. For any initial condition, the movement of targets will be on a limit cycle after some time. Here, the initial condition of targets are chosen as $(-2, -2)$, $(2, 3)$, $(3, -2)$, $(0, -4)$, $(-3, 2)$ to make sure no collision happens. Based on Fig. 3, the movement of targets will be on a limit-cycle and all targets rotate as well as keeping a desired distance from each other.

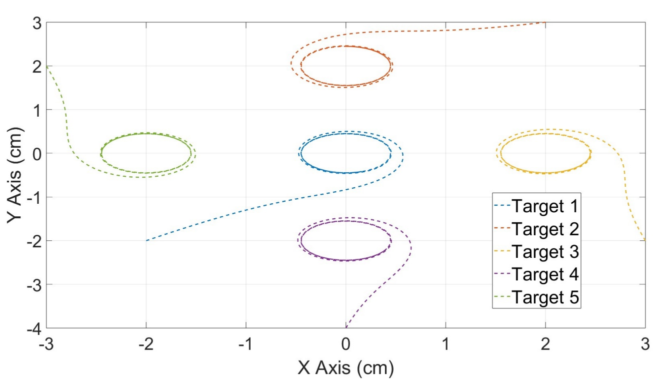

Fig. 3. The trajectories of targets' movement evolution

The initial conditions of the agents are $(0, 0)$, $(0, 0)$, $(0, 0)$, $(0, 0)$, $(0, 0)$. The design parameters of estimation section has been chosen as $\alpha = 1, \kappa = 1$, for all $i, \hat{j}$ and the communication delay is $d = 0.1$s. Note that the communication delay [18], which refers to the delay in data transmission from the transmitter to the receiver, is typically no greater than 0.01s. However, we consider here a larger delay over the actual range to assess the methodology's effectiveness.

The angular velocity $\mu_i(t) = 1$, and we choose $\xi_1 = 2$, $v = 1$ arbitrary.The control design parameters γ_1, γ_2, and γ_3 have been selected as 6.5, 1.5, and 14, respectively, based on the Theorems 3 and 4 and the stability analysis of this Theorems are omitted in this paper but the acceptable range for this parameters are in the stability analysis. These choices ensure the stability of the closed-loop system, confirming the establishment of the control.

Fig. 4 demonstrates how the agents have established a surrounding configuration in the two-dimensional plane. Fig. 5 provides a time-based visualization of the encirclement, where the vertical axis represents time. Fig. 6 demonstrates the convergence of the error signal for the desired polar radius to zero as time approaches infinity. Notably, the agents maintain a safe distance from one another throughout the rotation, ensuring there are no collisions between them. Fig. 7 illustrates the convergence of the error signal for the system to zero as time approaches infinity. The chattering phenomenon caused by control (30a) is clearly obvious in Fig. 5 and Fig. 6. Besides, Fig. 8 depicts the control input effort of all agents and demonstrates that the required input control has a bounded amplitude.

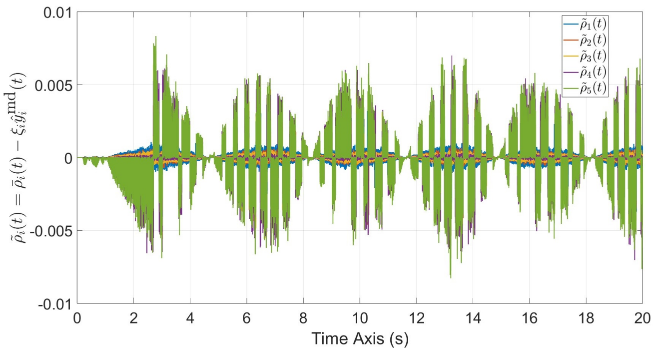

Fig. 6. The discrepancy between the measured relative desired polar radius and ξ_i times the estimated maximum distance of targets to the geometric center ($\tilde{\rho}_i(t) = \bar{\rho}_i(t) - \xi_i \hat{y}_i^{\mathrm{md}}(t)$)

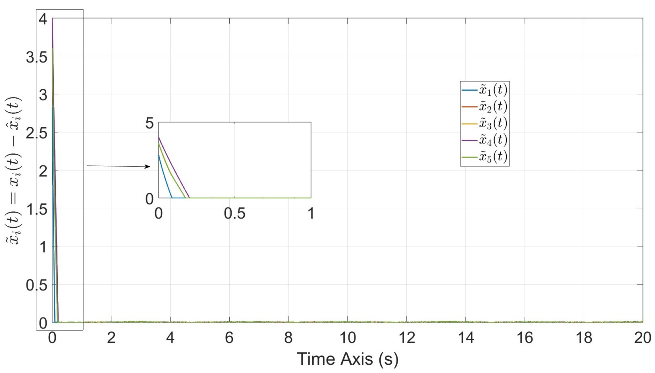

Fig. 7. The discrepancy between the $\boldsymbol{x}_i(t)$ and the relative desired location ($\tilde{\boldsymbol{x}}_{\boldsymbol{i}}(t) = \boldsymbol{x}_i(t) - \hat{\boldsymbol{x}}_i(t)$)

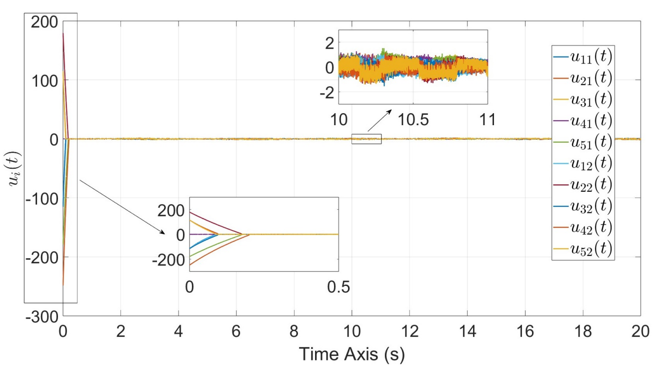

Fig. 8. The control input effort of agents

VI. Conclusion and Future works

This paper presented a novel distributed encirclement control for MASs that effectively tackles the challenge of communication delay. Extensive analysis showed that the proposed system can achieve its objective even under the influence of maximum constant communication delay. This represented a significant advancement compared to prior works, as it is the first time a control for dynamic encirclement with consideration of communication delay has been investigated. Simulation examples demonstrate that the primary control successfully achieved dynamic encirclement.

The impact of the chattering phenomena caused by the sign function is apparent in all the results. Resolving chattering phenomena will be the future work. Considering the definition of delay, further investigations could explore the impact of time-varying delay. Additionally, this study focused on first-order systems as the model, but it would be valuable to consider higher-order models of systems. The stability of the closed-loop system, based on simulation results, could be subject to more extensive analysis.

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