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Energy-based stabilisation and H_∞ robust stabilisation of stochastic non-linear systems

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Yan-Hong Liu[1] ✉, *Gui-Zhou Cao*[1], *Shu-Xia Tang*[2], *Xiu-Shan Cai*[3], *Jin-Zhu Peng*[1]

[1]*School of Electrical Engineering, Zhengzhou University, Zhengzhou 450001, People's Republic of China*
[2]*Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093, USA*
[3]*College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, People's Republic of China*
✉ *E-mail: liuyh@zzu.edu.cn*

Abstract: This study proposes a constructive stabilisation and H_∞ robust controller design method for stochastic non-linear systems from a novel dissipation analysis and energy point of view. First, the authors propose a sufficient condition for the dissipation of stochastic Hamiltonian systems and discuss the energy property of the systems, which will be used for the stability analysis and feedback controller design. Then, the authors show that the system is (asymptotically) stable in probability if it is (strictly) dissipative. By completing the Hamiltonian realisation of the stochastic non-linear systems, a feedback controller is proposed to stabilise the system under the condition of dissipation and zero state detectability. For stochastic non-linear systems subjected to external disturbances, an energy-based H_∞ controller was proposed by choosing the Hamiltonian function to construct a solution of Hamiltonian–Jacobi inequality. Finally, the effectiveness of the proposed method was illustrated via the inverted pendulum systems.

1 Introduction

There are various types of random disturbances in the physical systems, such as environmental changes, measurement noises, friction force and model uncertainty. The dynamics of the disturbed systems can be presented as stochastic non-linear systems [1]. However, random uncertainties can decrease the performance, such as the stability and robustness of the systems. So far, a lot of work has been carried out on the analysis and synthesis of stochastic non-linear systems. Deng *et al.* [2] and Mao [3] addressed some sufficient conditions for the stability and asymptotical stability in probability based on the stochastic Lyapunov function method and stochastic La Salle's invariant principle. Florchinger [4] constructed a state feedback stabilisation controller for stochastic non-linear systems via the control Lyapunov function method. Deng *et al.* [5] proposed a Backstepping procedure to stabilise stochastic non-linear systems with unknown covariance noises. Niu *et al.* [6] investigated the disturbance attenuation problem of stochastic non-linear systems subjected to external disturbances and proposed a solvability condition for Hamiltonian–Jacobi inequality. In [7–9], non-linear stochastic H_∞ controllers were constructed for stochastic non-linear systems by solving the Hamiltonian–Jacobi equations or inequalities.

In general, it is difficult to find a suitable Lyapunov function for the stability analysis and stabilisation of stochastic non-linear systems. Moreover, for the robust control of stochastic non-linear systems, it is also a difficult task to solve the Hamiltonian–Jacobi equation and inequality. Note that the passivity property of non-linear systems has a natural relationship with the stability and the storage function can be chosen as a solution of Hamiltonian–Jacobi inequality under some conditions, many researchers have made efforts to extend the passive theory of deterministic non-linear systems to stochastic non-linear systems. By utilising the passivity characters of stochastic systems, Florchinger [10] proposed some sufficient conditions for the asymptotical stability of the systems. Also, the global passivity-based stabilisation controllers were constructed in [11]. Lin *et al.* [12] investigated the problem of stochastic passivity, feedback passivity and stabilisation of stochastic non-linear systems. Ferreira *et al.* [13] proposed some sufficient conditions for the stability in probability and noise-to-state stability of large-scale non-linear stochastic systems by using the stochastic passivity properties of subsystems. Wu *et al.* [14] extended the dissipativity theory to the stochastic case and proposed some criteria for the stability analysis of stochastic non-linear systems. Recently, Rajpurohit *et al.* [15] discussed the dissipativity of controlled Markov diffusion processes and proposed some extended Kalman–Yakubovich–Popov conditions in terms of the drift and diffusion dynamics for characterising stochastic dissipativity via storage functions. However, how to construct a storage function to complete the passivity-based or dissipativity-based stability analysis and stabilisation control still largely remains open.

The Hamiltonian function method views the non-linear systems from the energy point of view and utilises their dissipation structure to complete the stability analysis and feedback controller design, see [16–18] and the references therein. One of the most important advantages of the Hamiltonian function method is that the Hamiltonian function can be chosen as a Lyapunov function candidate to perform stability analysis and to construct a solution for Hamiltonian–Jacobi inequality. Recently, Satoh *et al.* [19, 20] extended the results of deterministic Hamiltonian systems to stochastic non-linear Hamiltonian systems and proposed a stabilising controller for the full-actuated and under-actuated systems in the presence of persistent noise disturbances. In this study, we discuss the energy-based stabilisation and H_∞ control of stochastic non-linear systems by reformulating them as stochastic Hamiltonian systems. First, the dissipation property of stochastic Hamiltonian systems is analysed. The internal structure and energy property of the systems are discussed as well. Then, the feedback stabilisation controller for stochastic non-linear systems is constructed by completing their Hamiltonian realisation. For the stochastic non-linear systems with external disturbances, the L_2 gain analysis and energy-based H_∞ control are investigated. Finally, this study extends the results in [21] and proposes a H_∞ controller for inverted pendulum systems by transforming them into feedback equivalent stochastic Hamiltonian systems. Simulation results demonstrate the effectiveness of the proposed method.

The rest of the paper is organised as follows. Section 2 discusses the dissipation and energy property of stochastic Hamiltonian systems. Section 3 puts forward an energy-based stabilisation controller of stochastic non-linear systems. In Section 4, a robust H_∞ controller is constructed for stochastic non-linear systems by completing their Hamiltonian realisation. Section 5 investigates the energy-based robust control of inverted pendulum systems subjected to external disturbances to illustrate the effectiveness of the proposed method. Finally, Section 6 draws the conclusion.

2 Dissipation and energy property of stochastic Hamiltonian systems

The Hamiltonian model provides a suitable representation of many physical systems and can explicitly present the essential energy interconnection and dissipation of the system. In this section, we discuss the dissipativity of stochastic Hamiltonian systems and explore their energy properties.

Consider the following stochastic Hamiltonian systems presented in the sense of the Itô differential equation

$$\begin{cases} \mathrm{d}x = \left[(J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + g(x)u\right]\mathrm{d}t \\ \quad + g_w(x)\,\mathrm{d}w, \\ y = g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u(t)$, and $y(t) \in \mathbb{R}^m$ are the state, control input and output, respectively. The signal $w(t) \in \mathbb{R}^r$ is a standard Wiener process defined on a probability space $(\Omega, \mathscr{F}, \mathscr{P})$. The structure matrix $J(x) \in \mathbb{R}^{(n\times n)}$ is skew-symmetric and the dissipation matrix $R(x) \in \mathbb{R}^{(n\times n)}$ is symmetric and positive semi-definite. The continuous differentiable function $H(x)$ is the Hamiltonian function. $f(x)$, $g(x)$ and $g_w(x)$ are supposed to be the Borel measurable matrix with suitable dimensions. Assume that the input u is a $\mathbb{R}^m$-valued measurable function and satisfies $E\left[\int_0^t \| u^2(s) \|\mathrm{d}s\right] < \infty$ with respect to the measure $\mathscr{P}$, denoted by $E[\,\cdot\,]$. For more details about stochastic Hamiltonian system, see [19].

Definition 2.1: For the system (1), if the Hamiltonian function $H(x)$ satisfies

$$E[H(x(t_1))] - E[H(x(t_0))] \le E\left[\int_{t_0}^{t_1} \phi(u, y)\,\mathrm{d}t\right], \tag{2}$$

for any initial condition $x(t_0) = x_0, t_1 \ge t_0$, then the system is dissipative with respect to the supply rate $\phi(u, y)$ and $H(x)$ is the corresponding storage function.

Theorem 1: Consider the stochastic Hamiltonian system (1). Assume that the Hamiltonian function $H(x)$ is positive definite and the following inequality holds

$$\begin{aligned} &-\frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{g_w(x)^{\mathrm{T}}\frac{\partial^2 H(x)}{\partial x}g_w(x)\right\} \le 0, \end{aligned} \tag{3}$$

then the system (1) is dissipative. Moreover, the system is strictly dissipative if

$$\begin{aligned} &-\frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{g_w(x)^{\mathrm{T}}\frac{\partial^2 H(x)}{\partial x}g_w(x)\right\} < 0. \end{aligned} \tag{4}$$

Proof: Choose the Hamiltonian function $H(x)$ as the storage function and $y^{\mathrm{T}}u$ as the supply rate. According to (3), we have

$$\begin{aligned} \mathscr{L}H(x) = &-\frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{g_w(x)^{\mathrm{T}}\frac{\partial^2 H(x)}{\partial x}g_w(x)\right\} + y^{\mathrm{T}}u \\ &\le y^{\mathrm{T}}u, \end{aligned} \tag{5}$$

where $\mathscr{L}$ is the infinitesimal generator of the solution of (1). Letting $0 \le t_0 \le t_1$, we have

$$\begin{aligned} &E\left[\int_{t_0}^{t_1} \mathscr{L}H(x)\mathrm{d}s\right] - E\left[\int_{t_0}^{t_1} y^{\mathrm{T}}u\,\mathrm{d}t\right] \\ &= E\left[\int_{t_0}^{t_1} \mathrm{d}H(x)\mathrm{d}s\right] - E\left[\int_{t_0}^{t_1} y^{\mathrm{T}}u\,\mathrm{d}t\right] \\ &= E[H(x(t_1))] - E[H(x(t_0))] - E\left[\int_{t_0}^{t_1} y^{\mathrm{T}}u\,\mathrm{d}t\right] \\ &\le 0, \end{aligned} \tag{6}$$

so the system (1) is dissipative. Moreover, if (4) holds, then $\mathscr{L}H(x) < y^{\mathrm{T}}u$ and the system is strictly dissipative. □

Remark 1: Theorem 1 extends from the (strict) dissipation of deterministic Hamiltonian systems. Actually, if we decompose the noise $w(t)$ from the system, i.e. $g_w(x) = 0$, the system reformulates to the deterministic Hamiltonian systems

$$\begin{cases} \dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + g(x)u, \\ y = g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}, \end{cases} \tag{7}$$

and (3) and (4) are reformed as

$$-\left(\frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x}\right) \le 0 \quad \text{and} -\left(\frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x}\right) < 0,$$

respectively (or $R(x) \ge 0(>0)$ equally), which are specifically the sufficient conditions of (strict) dissipation of deterministic Hamiltonian systems.

Now we analyse the energy property of the stochastic Hamiltonian systems. The derivative of $H(x)$ along the trajectories of system (1) can be written as

$$\mathrm{d}H(x) = \mathscr{L}H(x) + \frac{\partial H(x)}{\partial x}g_w(x)\,\mathrm{d}w, \quad P - \mathrm{a.s.} \tag{8}$$

Taking integration of (6), we have

$$\begin{aligned} &H(x(t)) - H(x(0)) \\ &= \int_0^t \mathscr{L}H(x(s))\mathrm{d}s + \int_0^t \frac{\partial H(x)}{\partial x}g_w(x)\,\mathrm{d}w, \quad P - \mathrm{a.s.} \end{aligned} \tag{9}$$

Substituting (5) into the above equation and calculating the expectation of both sides, we have the following energy balance equation of stochastic Hamiltonian systems:

$$\begin{aligned} &\underbrace{E\{H(x(t))\} - E\{H(x(0))\}}_{\text{change of total energy}} \\ &= \underbrace{E\left\{-\int_0^t \frac{\partial^{\mathrm{T}} H(x)}{\partial x}R(x)\frac{\partial H(x)}{\partial x}\mathrm{d}s\right\}}_{\text{energy dissipation}} + \underbrace{E\left\{\int_0^t y^{\mathrm{T}}u\mathrm{d}s\right\}}_{\text{energy injection}} \\ &+\underbrace{E\left\{\int_0^t \frac{1}{2}\mathrm{Tr}\left\{g_w(x)^{\mathrm{T}}\frac{\partial^2 H(x)}{\partial x}g_w(x)\right\}\mathrm{d}s\right\}}_{\text{energy generated by noises}}. \end{aligned} \tag{10}$$

For the deterministic Hamiltonian system (7), the time derivative of the Hamiltonian function $H(x)$ is

$$\dot{H}(x) = -\frac{\partial^{\mathrm{T}} H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} + y^{\mathrm{T}} u \tag{11}$$

and the energy balance equation of the deterministic Hamiltonian systems is

$$\underbrace{H(x(t)) - H(x(0))}_{\text{change of total energy}} = \underbrace{-\int_0^t \frac{\partial^{\mathrm{T}} H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} \mathrm{d}s}_{\text{energy dissipation}} + \underbrace{\int_0^t y^{\mathrm{T}} u \, \mathrm{d}s}_{\text{energy injection}}. \tag{12}$$

Comparing (7) with (10), we can see that for stochastic Hamiltonian systems, the noises can be viewed as sources in the system and the term

$$E\left\{\int_0^T \frac{1}{2} \mathrm{Tr}\left\{g_w(x)^{\mathrm{T}} \frac{\partial^2 H(x)}{\partial x} g_w(x)\right\} \mathrm{d}s\right\}$$

represents the energy generated by the noises. So, the stochastic Hamiltonian system is a more general open system with internal energy transformation (i.e. $(\partial^{\mathrm{T}} H(x)/\partial x) J(x) (\partial H(x)/\partial x) = 0$), energy dissipation, energy generation and external energy injection from input–output ports. In the next sections, by reformulating the considered stochastic non-linear systems into their equivalent stochastic Hamiltonian systems, we will take advantage of the energy property and the specified structure of the system to complete the stabilisation and H_∞ controller design.

3 Energy-based stabilisation of stochastic non-linear systems

Consider the following stochastic non-linear systems presented by the Itô differential equation

$$\begin{cases} \mathrm{d}x = f(x)\,\mathrm{d}t + g(x) u \,\mathrm{d}t + g_w(x)\,\mathrm{d}w, \\ y = h(x), \end{cases} \tag{13}$$

where $x \in \mathbb{R}^n$ and $u, y \in \mathbb{R}^m$ are the state, control input and measured output, respectively. $w(t) \in \mathbb{R}^r$ is a standard Wiener process. $f(x)$, $g(x)$ and $g_w(x)$ are Borel measurable matrices with suitable dimensions.

Definition 3.1: Suppose there exists a continuously differentiable function $H(x)$ and structure $J^{\mathrm{T}}(x) = -J(x)$ and $R(x) \geq 0$ such that the stochastic non-linear system (13) can be reformulated as the stochastic Hamiltonian system (1), then (1) is called a Hamiltonian realisation of (13).

Theorem 2: Suppose the stochastic non-linear system (13) can be dissipative Hamiltonian realised as (1) with respect to a positive definite continuously differentiable Hamiltonian function $H(x)$, then the trivial solution of the system (13) is stable in probability. Also, the strict dissipation implies asymptotical stability in probability.

Proof: Choosing the Hamiltonian function $H(x)$ as a stochastic Lyapunov function and noticing that

$$\mathscr{L}(H(x)) = -\frac{\partial^{\mathrm{T}} H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} + \frac{1}{2} \mathrm{Tr}\left\{g_w(x)^{\mathrm{T}} \frac{\partial^2 H(x)}{\partial x} g_w(x)\right\}, \tag{14}$$

we can get $LH \leq 0$ if the system is dissipative and $LH < 0$ if it is strict dissipative. According to the stability theory of stochastic non-linear systems, we have the results directly. □

For the feedback stabilisation of stochastic non-linear systems, we have the following result.

Theorem 3: For the stochastic non-linear system (13), suppose the system can be dissipative Hamiltonian realised as (1). If the following conditions hold:

(i) the Hamiltonian function $H(x)$ is positive definite continuous differentiable with the origin being its minimum point;
(ii) the system is zero state detectable,

then the system (13) can be stabilised by

$$u = -K(x) g^{\mathrm{T}}(x) \frac{\partial H(x)}{\partial x}, \tag{15}$$

where $K(x) > 0$ is the feedback gain.

Proof: Choose $V(x) = H(x) - H(0)$ as a candidate Lyapunov function. For the closed loop system, we have

$$\begin{aligned} \mathscr{L}V = & -\frac{\partial^{\mathrm{T}} H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} + \frac{1}{2} \mathrm{Tr}\left\{g_w(x)^{\mathrm{T}} \frac{\partial^2 H(x)}{\partial x} g_w(x)\right\} \\ & -\frac{\partial^{\mathrm{T}} H(x)}{\partial x} g(x) K(x) g^{\mathrm{T}}(x) \frac{\partial H(x)}{\partial x} \\ \leq & -\frac{\partial^{\mathrm{T}} H(x)}{\partial x} g(x) K(x) g^{\mathrm{T}}(x) \frac{\partial H(x)}{\partial x}. \end{aligned}$$

Note $K(x) > 0$, the points that make $\mathscr{L}V = 0$ must satisfy $g^{\mathrm{T}}(x)(\partial H(x)/\partial x) = h(x) = 0$. According to the zero state detectablility of the system, the state $x(t)$ tends to 0 in probability. □

Remark 2: Similar to the energy-based stabilisation controller design of non-linear systems, the feedback controller (15) stabilises the stochastic non-linear system by injecting damp into the system. If the stochastic Hamiltonian realisation system is not dissipative, one can choose the feedback gain large enough to make the closed loop system dissipative and asymptotically stable in probability.

Remark 3: Compared with the other passivity-based stabilisation of stochastic non-linear systems (such as Florchinger [11, 13], Lin *et al.* [12], Wu *et al.* [14], etc.), where it is generally very difficult to seek a suitable storage function to complete the stability analysis and controller design, this study proposes a constructive method by transforming the considered stochastic non-linear systems to its equivalent Hamiltonian formulation and use the internal structure and energy property to construct a stabilisation controller. The Hamiltonian function can be chosen as a Lyapunov candidate to complete the stability analysis.

Remark 4: The key to reformulate the system (13) as a stochastic Hamiltonian system is to seek a continuous differentiable function $H(x)$, a positive semi-definite matrix $R(x)$ and a skew-symmetric matrix $J(x)$ such that $f(x) = (J(x) - R(x))(\partial H(x)/\partial x)$. There are many methods to complete the Hamiltonian realisation [22]. For mechanical systems and power systems, the inherit energy property is generally utilised to complete their Hamiltonian realisation.

4 Energy-based H_∞ robust control of stochastic non-linear systems

In this section, we consider the robust control of stochastic non-linear systems subjected to external disturbances by completing the Hamiltonian realisation.

Consider the following uncertain stochastic non-linear system:

$$\begin{cases} \mathrm{d}x = f(x)\,\mathrm{d}t + g_v(x)v\,\mathrm{d}t + g_w(x)\,\mathrm{d}w, \\ z = h_1(x), \end{cases} \quad (16)$$

where $v \in \mathbb{R}^r$ is the unknown bounded disturbance signal, $g_v \in \mathbb{R}^{n \times r}$ is a matrix valued function and $z \in \mathbb{R}^q$ is an estimation variable.

Theorem 4: For a given disturbance attenuation level $\gamma > 0$, suppose there exists a non-negative differentiable function $V(x) \ge 0$ $(V(0) = 0)$ such that the following Hamiltonian–Jacobi inequality holds

$$\frac{\partial^{\mathrm{T}} V}{\partial x} f + \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} V}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial V}{\partial x} + \frac{1}{2} h_1^{\mathrm{T}} h_1 + \frac{1}{2}\mathrm{Tr}\left\{ g_w^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2} g_w \right\} \le 0, \quad (17)$$

the L_2 gain of the system (16) is not more than γ, i.e.

$$E\left(\int_0^T \| z \|^2 \mathrm{d}t\right) \le E\left(\gamma^2 \int_0^T \| v \|^2 \mathrm{d}t\right), \quad \forall v \in L_2[0, T]. \quad (18)$$

Proof: If $V(x)$ is a differentiable scalar function which satisfies (17), then we have

$$\begin{aligned} \mathscr{L}V &= \frac{\partial^{\mathrm{T}} V}{\partial x} f + \frac{\partial^{\mathrm{T}} V}{\partial x} g_v v + \frac{1}{2}\mathrm{Tr}\left\{ g_w^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2} g_w \right\} \\ &\le -\frac{1}{2} h_1^{\mathrm{T}} h_1 - \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} V}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial V}{\partial x} + \frac{\partial^{\mathrm{T}} V}{\partial x} g_v v \\ &\quad -\frac{1}{2}\gamma^2 v^{\mathrm{T}} v + \frac{1}{2}\gamma^2 v^{\mathrm{T}} v \\ &= \frac{1}{2}\{\gamma^2 \| v \|^2 - \| z \|^2\} - \frac{1}{2}\left\| \gamma v - \frac{1}{\gamma} g_v^{\mathrm{T}} \frac{\partial^{\mathrm{T}} V}{\partial x} \right\|^2. \end{aligned}$$

Note that

$$\left\| \gamma v - \frac{1}{\gamma} g_v^{\mathrm{T}} \frac{\partial^{\mathrm{T}} H}{\partial x} \right\|^2 \ge 0,$$

we get

$$\mathscr{L}V \le \frac{1}{2}\{\gamma^2 \| v \|^2 - \| z \|^2\}. \quad (19)$$

By the It ô's formula

$$E\{V(T)\} - E\{V(0)\} \le E\left\{ \frac{1}{2}\int_0^T (\gamma^2 \| v \|^2 - \| z \|^2)\,\mathrm{d}t \right\}. \quad (20)$$

Since $V(x) \ge 0$ and $V(0) = 0$, the inequality (20) is equivalent to (18).

Now we consider the H_∞ control of following stochastic non-linear systems subjected to external disturbances

$$\begin{cases} \mathrm{d}x = f(x)\,\mathrm{d}t + g(x)u\,\mathrm{d}t + g_v(x)v\,\mathrm{d}t + \tilde{g}(x)\,\mathrm{d}w, \\ z = h_1(x), \\ y = h_2(x), \end{cases} \quad (21)$$

where $v \in \mathbb{R}^s$ and $z \in \mathbb{R}^m$ are, respectively, the unknown bounded disturbance signal and the estimation variable. $g_v(x)$ and $\tilde{g}(x)$ are matrix-valued functions with proper dimensions. The other variables can be referred to (11).

The objective of the H_∞ control is to construct a state feedback control law $u(x) = \alpha(x)$ with $\alpha(0) = 0$ such that the L_2 gain of the closed loop system is not more than the given disturbance attenuation value γ and the homogeneous closed loop system is asymptotically stable in probability. □

Theorem 5: Consider the system (21). Suppose it can be a dissipative Hamiltonian realised as

$$\begin{cases} \mathrm{d}x = \left[(J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + g(x)u + g_v(x)v \right]\mathrm{d}t \\ \qquad + \tilde{g}(x)\,\mathrm{d}w, \\ z = r(x)g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}, \\ y = g_v^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}, \end{cases} \quad (22)$$

where $r(x)$ is a weighted matrix with a full column rank. Suppose the following conditions hold:

(i) the Hamiltonian function $H(x)$ is positive definite continuous differentiable with the origin being its minimum point;
(ii) the system is zero state detectable;
(iii) $Kg(x)g^{\mathrm{T}}(x) + \dfrac{1}{2\gamma^2}\left[g(x)g^{\mathrm{T}}(x) - g_v(x)g_v^{\mathrm{T}}(x) \right] \ge 0$.

Then for the given disturbance attenuation level $\gamma > 0$, a H_∞ controller can be constructed as

$$u(x) = -\left[K + \frac{1}{2} r^{\mathrm{T}}(x)r(x) + \frac{1}{2\gamma^2} \right] g^{\mathrm{T}}(x)\frac{\partial H}{\partial x}, \quad (23)$$

where $K > 0$.

Proof: First, we will show the L_2 gain of the closed loop system is not more than γ. Substituting the feedback controller (23) into system (22), we have

$$\begin{cases} \mathrm{d}x = \left[\left(J - R - g\left[K + \dfrac{1}{2} r^{\mathrm{T}}(x)r(x) + \dfrac{1}{2\gamma^2} \right] g^{\mathrm{T}} \right)\dfrac{\partial H(x)}{\partial x} \right]\mathrm{d}t \\ \quad + g_v(x)v\,\mathrm{d}t + \tilde{g}(x)\,\mathrm{d}w \\ \quad = \tilde{f}(x)\,\mathrm{d}t + g_v(x)v\,\mathrm{d}t + \tilde{g}(x)\,\mathrm{d}w, \\ z = g^{\mathrm{T}}(x)\dfrac{\partial H(x)}{\partial x}. \end{cases} \quad (24)$$

Along the trajectories of the system (24), we get

$$\begin{aligned} &\frac{\partial^{\mathrm{T}} H}{\partial x}\tilde{f} + \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial H}{\partial x} + \frac{1}{2} h_1^{\mathrm{T}} h_1 \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial x^2}\tilde{g} \right\} \\ &= \frac{\partial^{\mathrm{T}} H}{\partial x}\left(J - R - g\left[K + \frac{1}{2} r^{\mathrm{T}}(x)r(x) + \frac{1}{2\gamma^2} \right] g^{\mathrm{T}} \right)\frac{\partial H}{\partial x} \\ &\quad + \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial H}{\partial x} + \frac{1}{2}\frac{\partial^{\mathrm{T}} H}{\partial x} g r^{\mathrm{T}}(x)r(x)g^{\mathrm{T}}\frac{\partial H}{\partial x} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2}\tilde{g} \right\} \\ &= -\frac{\partial^{\mathrm{T}} H}{\partial x} R \frac{\partial H}{\partial x} + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 V}{\partial x^2}\tilde{g} \right\} + \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial H}{\partial x} \\ &\quad - \left(K + \frac{1}{2\gamma^2} \right)\frac{\partial^{\mathrm{T}} H}{\partial x} g g^{\mathrm{T}} \frac{\partial H}{\partial x} \\ &\le 0. \end{aligned}$$

□

According to Theorem 4, the L_2 gain of the closed loop system is not more than γ.

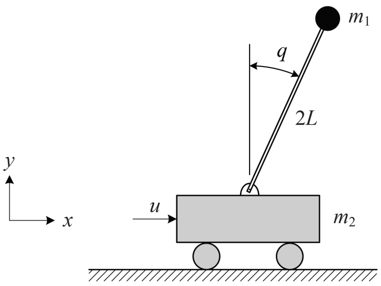

Fig. 1 *Inverted pendulum system*

Next, we will show the homogeneous part of the closed loop system is asymptotically stable in probability. Choosing $V(x) = H(x) - H(0) > 0$ as a Lyapunov function, we have

$$\begin{aligned}\mathscr{L}V = &-\frac{\partial^{\mathrm{T}} H}{\partial x}R(x)\frac{\partial H}{\partial x} + \frac{1}{2}\mathrm{Tr}\left\{\tilde{g}^{\mathrm{T}}(x)\frac{\partial^2 H}{\partial x^2}\tilde{g}(x)\right\}\\ &-\frac{\partial^{\mathrm{T}} H}{\partial x}g(x)\left[K + \frac{1}{2}r^{\mathrm{T}}(x)r(x) + \frac{1}{2\gamma^2}\right]g^{\mathrm{T}}(x)\frac{\partial H}{\partial x}\\ \le &-\frac{\partial^{\mathrm{T}} H}{\partial x}Kg(x)g^{\mathrm{T}}(x)\frac{\partial H}{\partial x} - \frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x}gg^{\mathrm{T}}\frac{\partial H}{\partial x}\\ &+\frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x}g_v g_v^{\mathrm{T}}\frac{\partial H}{\partial x} - \frac{1}{2}\frac{\partial^{\mathrm{T}} H}{\partial x}gr^{\mathrm{T}}(x)r(x)g^{\mathrm{T}}\frac{\partial H}{\partial x}\\ &-\frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x}g_v g_v^{\mathrm{T}}\frac{\partial H}{\partial x}\\ \le &-\frac{1}{2\gamma^2}\frac{\partial^{\mathrm{T}} H}{\partial x}g_v(x)g_v^{\mathrm{T}}(x)\frac{\partial H}{\partial x}\\ \le &\ 0.\end{aligned}$$

So, we have $\mathscr{L}V = 0$ if and only if $y = g_v^{\mathrm{T}}(x)(\partial H/\partial x) = 0$. According to the condition (ii), the closed loop system with $v = 0$ is asymptotically stable in probability.

Remark 5: It can be seen from Theorem 4 that the Hamiltonian function and internal structure can be utilised to construct a robust controller. For a given uncertain stochastic non-linear system, one can follow three steps to obtain an energy-based H_∞ controller: (i) to re-formulate the considered stochastic non-linear system as an equivalent stochastic Hamiltonian formulation(24); (ii) to verify the dissipativity of the system. If the system is not dissipative, one can design a feedback controller to inject damp and make the closed loop system dissipative (as shown in Remark 3, Section 3); (iii) to select the Hamiltonian function as a solution of the Hamiltonian–Jacobi inequality and construct an energy-based H_∞ controller (23) by utilising the internal structure of the system.

5 Energy-based H_∞ control of uncertain inverted pendulum systems

5.1 Stochastic dynamic modelling and Hamiltonian realisation

Consider the inverted pendulum system shown in Fig. 1.

The dynamics of the system can be written as [22, 23]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)u, \tag{25}$$

where q is the angle between the pendulum rod and the vertical direction

$$M(q) = L\left(\frac{4}{3} - \frac{m_1\cos^2 q}{m_1 + m_2}\right)$$

is the inertia matrix, where L, m_1 and m_2 are a half of the length of the rod, the mass of the pendulum and the mass of the cart, respectively

$$C(q,\dot{q}) = \frac{m_1 L\dot{q}\sin(2q)}{2(m_1 + m_2)}$$

is the Coriolis/centripetal matrix. $G(q) = -g\sin q$ is the gravity matrix, where $g = 9.8\,\mathrm{m/s^2}$ is the acceleration of the gravity. $B(q) = (\cos q/(m_1 + m_2))$ is the control input coefficient. u is the force acting on the cart.

Denote $p = M(q)\dot{q}$ as the generalised momentum, then the system (25) can be reformed as

$$\begin{cases}\dot{q} = M^{-1}(q)p,\\ \dot{p} = -\dfrac{1}{2}p^2\left(\dfrac{\partial M^{-1}(q)}{\partial q}\right) + B(q)u - G(q).\end{cases} \tag{26}$$

Incorporating the possibly existing disturbances from the input channel and stochastic disturbances from the external environment, the dynamics of the inverted pendulum can be further modelled by the following stochastic system:

$$\begin{cases}\mathrm{d}q = M^{-1}(q)p\,\mathrm{d}t,\\ \mathrm{d}p = -\dfrac{1}{2}p^2\left(\dfrac{\partial M^{-1}(q)}{\partial q}\right)\mathrm{d}t + B(q)u\,\mathrm{d}t - G(q)\,\mathrm{d}t\\ \quad +B(q)v\,\mathrm{d}t + p\,\mathrm{d}w,\end{cases} \tag{27}$$

where v represents the disturbance from the input channel and w is the stochastic disturbance caused by the vibration of the system, which is generally directly related to the speed and mass.

To design the robust controller for the inverted pendulum system, we need to first reformulate the system as an equivalent stochastic Hamiltonian system. Assume q^* is the desired position and choose the Hamiltonian function

$$H(p,q) = \frac{p^2}{2M(q)} + (q - q^*)^2. \tag{28}$$

The Jacobian matrix of $H(p,q)$ is

$$\begin{bmatrix}\dfrac{\partial H}{\partial q}\\ \dfrac{\partial H}{\partial p}\end{bmatrix} = \begin{bmatrix}\dfrac{1}{2}p^2\dfrac{\partial M^{-1}(q)}{\partial q} + 2(q - q^*)\\ M^{-1}(q)p\end{bmatrix}. \tag{29}$$

The calculation shows that the inverted pendulum cannot be transformed to the Hamiltonian system directly with the chosen Hamiltonian function. We need to design a feedback controller to complete the Hamiltonian realisation and stabilise the system.

Consider the following feedback controller:

$$u = B^{-1}(q)\left[G(q) - 2(q - q^*) - K_d M^{-1}(q)p\right] + \bar{u}, \tag{30}$$

where $\bar{u}$ is the new control input to be designed and $K_d > 2L/3$ is an adjustable constant. The closed loop system is

$$\begin{cases}\mathrm{d}q = M^{-1}(q)p\,\mathrm{d}t,\\ \mathrm{d}p = -\dfrac{1}{2}p^2\left(\dfrac{\partial M^{-1}(q)}{\partial q}\right)\mathrm{d}t + B(q)\bar{u}\,\mathrm{d}t\\ \quad +B(q)\left[G(q) - 2(q - q^*) - K_d M^{-1}(q)p\right]\mathrm{d}t\\ \quad -G(q)\,\mathrm{d}t + B(q)v\,\mathrm{d}t + p\,\mathrm{d}w,\end{cases} \tag{31}$$

which can be re-written as the following standard stochastic Hamiltonian system:

$$\begin{aligned}\mathrm{d}x = &\left(J(x) - R(x)\right)\frac{\partial H}{\partial x}\mathrm{d}t + g(x)\bar{u}\,\mathrm{d}t\\ &+g_v(x)v\,\mathrm{d}t + \tilde{g}\,\mathrm{d}w,\end{aligned} \tag{32}$$

where $x = [q, p]^{\mathrm{T}}, g(x) = g_v(x) = [0, B(q)]^{\mathrm{T}}$

$$J(x) = -J^{\mathrm{T}}(x) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad R(x) = \begin{bmatrix} 0 & 0 \\ 0 & K_d \end{bmatrix} \geq 0,$$
$$\tilde{g} = \begin{bmatrix} 0 & 0 \\ 0 & p \end{bmatrix}.$$

Moreover, the infinitesimal generator of $H(x)$ is

$$\begin{aligned} \mathscr{L}H &= \begin{bmatrix} \frac{\partial H}{\partial q} & \frac{\partial H}{\partial p} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -K_d \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}}(x) \frac{\partial^2 H}{\partial x^2} \tilde{g}(x) \right\} \\ &= -K_d \left(\frac{\partial H}{\partial p} \right)^2 + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}}(x) \frac{\partial^2 H}{\partial x^2} \tilde{g}(x) \right\} \\ &= -K_d \left(M^{-1}(q) p \right)^2 + \frac{1}{2} p^2 M^{-1}(q) \\ &\leq \frac{p^2}{L} \frac{3(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} \left(-\frac{3K_d}{4L} + \frac{1}{2} \right). \end{aligned} \tag{33}$$

Note that

$$K_d > \frac{2}{3}L \quad \text{and} \quad \frac{3(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} > 0,$$

we have $\mathscr{L}H \leq 0$. So, the Hamiltonian formulation (32) is dissipative.

5.2 H_∞ robust controller design

Now we consider the H_∞ control of the system with the estimation variable

$$\begin{aligned} z &= r g^{\mathrm{T}}(x) \frac{\partial H(x)}{\partial x} = r B(q) \dot{q} \\ &= \frac{3 r p \cos(q)}{4(m_1 + m_2) - 3m_1 \cos^2(q)}, \end{aligned} \tag{34}$$

where $r > 0$ is the weight parameter. The estimation signal z is meaningful in that it represents the error between the angle speed and its desired value.

It is obvious that the Hamiltonian function $H(q, p)$ is positive definite and achieves strict minimum at $(q^*, 0)$. According to (23) in Theorem 5, for the given disturbance attenuation value $\gamma > 0$, the H_∞ controller can be constructed as

$$\bar{u} = -\left(K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2} \right) g^{\mathrm{T}}(x) \frac{\partial H}{\partial x}, \tag{35}$$

where $K > 0$.

The corresponding closed loop system is

$$\begin{aligned} \mathrm{d}x &= \left\{ \begin{bmatrix} 0 & 1 \\ -1 & -K_d \end{bmatrix} - g\left(K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2} \right) g^{\mathrm{T}} \right\} \frac{\partial H}{\partial x} \mathrm{d}t \\ &\quad + g_v(x) v \, \mathrm{d}t + \tilde{g} \, \mathrm{d}w \\ &= \bar{f} \, \mathrm{d}x + g_v(x) v \, \mathrm{d}t + \tilde{g} \, \mathrm{d}w. \end{aligned}$$

It can be verified that for the given γ

$$\begin{aligned} &\frac{\partial^{\mathrm{T}} H}{\partial x} \bar{f} + \frac{1}{2\gamma^2} \frac{\partial^{\mathrm{T}} H}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial H}{\partial x} + \frac{1}{2} z^{\mathrm{T}} z \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial x^2} \tilde{g} \right\} \\ &= -\frac{\partial^{\mathrm{T}} H}{\partial x} R \frac{\partial H}{\partial x} - \left(K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2} \right) \frac{\partial^{\mathrm{T}} H}{\partial x} g g^{\mathrm{T}}(x) \frac{\partial H}{\partial x} \\ &\quad + \frac{1}{2\gamma^2} \frac{\partial^{\mathrm{T}} H}{\partial x} g_v g_v^{\mathrm{T}} \frac{\partial H}{\partial x} + \frac{1}{2} \frac{\partial^{\mathrm{T}} H}{\partial x} g r^2 g^{\mathrm{T}} \frac{\partial H}{\partial x} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial x^2} \tilde{g} \right\} \\ &= -\frac{\partial^{\mathrm{T}} H}{\partial x} R \frac{\partial H}{\partial x} + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial x^2} \tilde{g} \right\} - K \frac{\partial^{\mathrm{T}} H}{\partial x} g g^{\mathrm{T}} \frac{\partial H}{\partial x} \\ &\leq \frac{p^2}{L} \frac{3(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} \left(-\frac{3K_d}{4L} + \frac{1}{2} \right) \\ &\quad - K \left(M^{-1}(q) p \frac{\cos(q)}{m_1 + m_2} \right)^2 \\ &\leq 0. \end{aligned}$$

So, the L_2 gain of the closed loop system is not more than γ.

Moreover, the infinitesimal generator of $H(x)$ along the trajectories of the closed loop system in the absence of v is

$$\begin{aligned} \mathscr{L}H &= \frac{\partial^{\mathrm{T}} H}{\partial x} \bar{f} + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial^2 x} \tilde{g} \right\} \\ &= -K_d \left(\frac{\partial H}{\partial p} \right)^2 + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}^{\mathrm{T}} \frac{\partial^2 H}{\partial^2 x} \tilde{g} \right\} \\ &\quad - \left(K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2} \right) \frac{\partial^{\mathrm{T}} H}{\partial x} g g^{\mathrm{T}} \frac{\partial H}{\partial x} \\ &\leq \frac{p^2}{L} \frac{3(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} \left(-\frac{3K_d}{4L} + \frac{1}{2} \right) \\ &\quad - (K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2}) \left(M^{-1}(q) p \frac{\cos(q)}{m_1 + m_2} \right)^2 \\ &\leq \frac{p^2}{L} \frac{3(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} \left(-\frac{3K_d}{4L} + \frac{1}{2} \right) \\ &\leq 0. \end{aligned}$$

Thus, the homogeneous system is stable in probability. Moreover, we have $\mathscr{L}H = 0$ only and if only

$$\frac{p^2(m_1 + m_2)}{4(m_1 + m_2) - 3m_1 \cos^2(q)} = 0$$

and the later means $p = 0$ and $q = q^*$, which is exactly the desired equilibrium point. According to LaSalle's invariance principle of stochastic non-linear systems, the homogeneous closed loop system is asymptotically stable in probability.

Integrating the above discussions, the H_∞ robust controller for the inverted pendulum system can be constructed as

$$\begin{aligned} u &= B^{-1}(q) \left[G(q) - 2(q - q^*) - K_d M^{-1}(q) p \right] \\ &\quad - \left(K + \frac{1}{2}r^2 + \frac{1}{2\gamma^2} \right) \frac{\dot{q} \cos q}{m_1 + m_2}. \end{aligned} \tag{36}$$

5.3 Simulation

To verify the effectiveness of the proposed control scheme, we simulate the inverted pendulum system under the following parameters: $m_1 = 0.1\,\mathrm{kg}$, $m_2 = 1\,\mathrm{kg}$ and $L = 0.5\,\mathrm{m}$. The weighted parameter $r = 1$ and the feedback gain $K_d = 1$, $K = 1$. The desired angle position is $q^* = \pi/6$. The external disturbance $v = \cos q$ and the disturbance attenuation value $\gamma = 1$.

The following cases are considered during the simulation:

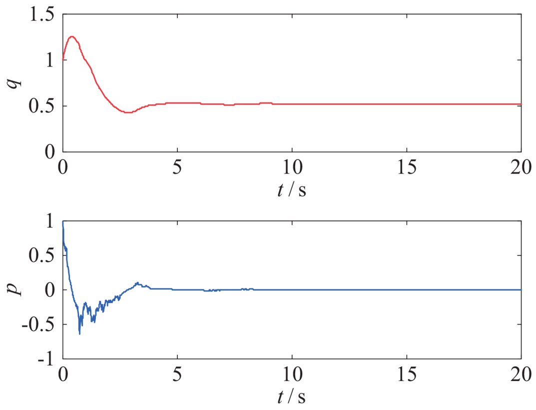

Fig. 2 *Responses of the inverted pendulum system ($v = 0$) under the feedback dissipation controller (30)*

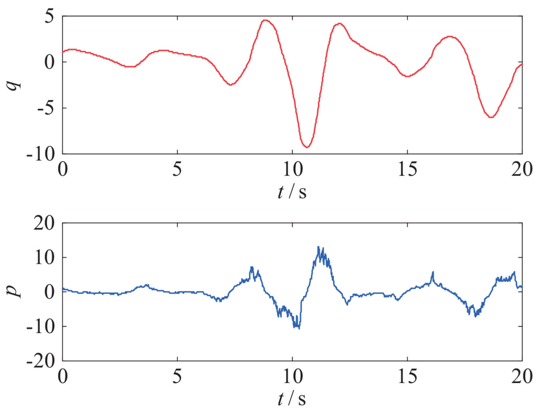

Fig. 3 *Responses of the inverted pendulum system ($v = \cos q$) under the feedback dissipation controller (30)*

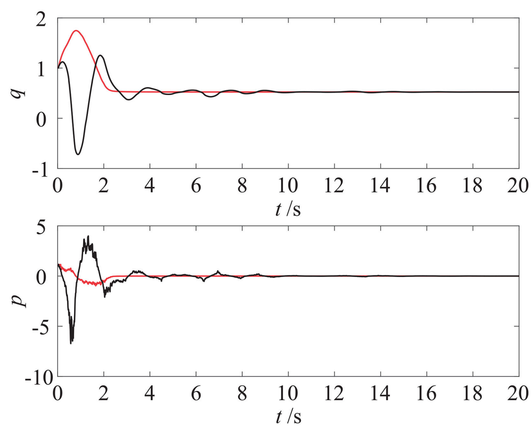

Fig. 4 *Responses of the inverted pendulum system ($v = \cos q$) under the energy-based robust controller (36) and filtered feedback linearisation controller (37), respectively*

*Case I:*The inverted pendulum system (without external disturbance, i.e. $v = 0$) under the feedback dissipation controller (30).

*Case II:*The inverted pendulum system (with external disturbance $v = \cos q$) under the dissipation controller (30).

*Case III:*The inverted pendulum system (with external disturbance $v = \cos q$) under the energy-based H_∞ controller (36) and the following filtered feedback linearisation controller [24], respectively

$$u_L = B^{-1}(q)[G(q) + C(q, \dot{q})\dot{q}] - K_{L1}(q - q^*) - K_{L2}p, \qquad (37)$$

where $K_{L1} = 12, K_{L2} = 3$.

The simulation results are shown in Figs. 2–4. In Fig. 4, the red line indicates the response of the system under the energy-based robust controller (36) and the black line shows the response under the filtered feedback linearisation controller (37).

From Fig. 2 we can see that when there are no external disturbances ($v = 0$), the feedback dissipation controller (30) can drive the inverted pendulum system stable, while in Fig. 3 we can see that when the system is subjected to external disturbance v, it oscillates intensely. Fig. 4 shows that the proposed energy-based H_∞ controller (36) and the filtered feedback linearisation controller (37) can both stabilise the system subjected to external disturbance from the input channel and stochastic disturbances from an external environment. However, compared with the feedback linearisation controller (37), there is a smaller oscillation and a shorter adjustment time under the H_∞ robust controller (36), which verifies the effectiveness of the proposed control method.

6 Conclusion

This study investigates the dissipation, stabilisation and H_∞ control of stochastic non-linear systems via the Hamiltonian system method. First, we propose a sufficient condition for the dissipation of the stochastic Hamiltonian systems. The energy dissipation, transformation, internal energy generation and external energy exchange property of the systems are explored as well. Then, by completing the Hamiltonian realisation of stochastic non-linear systems, a feedback stabilising controller is proposed by utilising the internal structure of the system. For stochastic non-linear systems subjected to the external disturbances, a H_∞ feedback controller is constructed with the Hamiltonian function being chosen to construct an explicit solution of Hamiltonian–Jacobi inequality. Finally, the robust control of inverted pendulum systems is discussed to verify the effectiveness of the proposed method. In the future research, we will investigate the parameter estimation and adaptive robust control of stochastic non-linear systems subjected to parameter uncertainties and external disturbances. It is also interesting to investigate the synchronisation of chaotic systems subjected to stochastic disturbances using an energy-based method.

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