Disturbance Estimation of a Wave PDE on a Time-varying Domain *

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Abstract

In this paper, we design an exponentially convergent disturbance observer for a wave PDE on a time-varying domain by using two boundary measurements $u(0,t), u_t(l(t),t)$. More specifically, two auxiliary PDEs are constructed to build the disturbance observer for tracking the external disturbance in the wave PDE. Exponential convergence of the disturbance estimation to the true disturbance value is proved by Lyapunov analysis, and all states in the observer are shown to be bounded once the original state $u(x,t)$ is bounded via designing control input.

1 Introduction

This paper considers a wave PDE on a time-varying domain with mismatched unknown disturbances, which can describe the axial vibration dynamics of a mining cable elevator with the disturbance at the cage. For the purpose of attenuating the disturbance in the feedback control design, the objective of this paper is to estimate the disturbance.

Different methods have been employed to deal with the disturbance in PDE systems in the past decades. Slide model control (SMC) is designed for heat, Euler-Bernoulli, and Schrödinger equations with boundary disturbances in [5],[8],[11],[13]. Adaptive control is applied to attenuate the harmonic disturbance with known frequencies in wave PDEs in [2],[3],[4].

An active disturbance rejection control (ADRC) method which was proposed by Han [10] has been used in PDE systems as well. The important step in ADRC is to estimate the unknown disturbance in time with measured outputs in a feasible way. The ADRC method has been verified theoretically and practically as a powerful method of dealing with disturbances in PDE systems by many research groups. In most of literatures about ADRC in PDE systems, the disturbance estimation converges asymptotically to the true value of the disturbance [7],[9]. The state feedback designs, using ADRC, of an one-dimensional anti-stable wave equation and a generalized version of the wave equation subject to matched disturbances $d(t)$ were presented in [6],[12]. The output feedback design of an one-dimensional anti-stable wave equation with a boundary disturbance $d(t)$ was developed in [7]. Boundary stabilization for a multi-dimensional wave equation with a boundary disturbance $d(t)$ was considered in [9]. An observer-based output feedback stabilising control for a wave PDE-ODE system with an external disturbance was proposed in [14].

Recently, output feedback stabilization of a wave equation with a matched external disturbance which is estimated by constructing two PDEs was considered in [1]. In this paper, we design a disturbance observer to track the actual disturbance with an exponentially convergent error in a wave PDE on a time-varying domain as follows:

$$u_{tt}(x,t) = u_{xx}(x,t), \tag{1.1}$$

$$u_x(0,t) = d(t), \tag{1.2}$$

$$u_x(l(t),t) = U(t), \tag{1.3}$$

where $u(x,t)$ is the PDE state. $d(t)$ is an unknown disturbance and $U(t)$ is the control input. The following assumptions are used.

ASSUMPTION 1. *The domain length $l(t)$ of the wave PDE is deceasing from its initial value L, i.e. $\dot{l}(t) \le 0$, and a lower bound $\underline{l} > 0$ exists on the domain length, s.t. $l(t) \ge \underline{l}, \forall t \ge 0$.*

ASSUMPTION 2. *The disturbance $d(t)$ is bounded by*

$$|d(t)| < \overline{D}, \tag{1.4}$$

where $\bar{D}$ is unknown and arbitrary.

The remainder of the paper is organized as follows. The design of the observer is presented in Section 2. The proof of the exponentially convergent estimate error of the observer is shown in Section 3 by Lyapunov analysis. The boundedness of all states in the observer is proved in Section 4. The conclusion is proposed in Section 5.

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2 Design of a Disturbance Observer

In this section, we design a disturbance estimator for the unknown disturbance in the wave PDE (1.1)-(1.3) on the time-varying domain by using the available measurements $u_t(l(t),t)$ $u(0,t)$:

$$\bar{d}_{tt}(x,t) = \bar{d}_{xx}(x,t), \tag{2.5}$$

$$\bar{d}(0,t) = u(0,t) - \bar{u}(0,t), \tag{2.6}$$

$$\bar{d}_x(l(t),t) = -a_1\bar{d}_t(l(t),t). \tag{2.7}$$

where $\bar{u}(x,t)$ satisfies the following system:

$$\bar{u}_{tt}(x,t) = \bar{u}_{xx}(x,t), \tag{2.8}$$

$$\bar{u}_x(0,t) = 0, \tag{2.9}$$

$$\bar{u}_x(l(t),t) = U(t) + a_1(u_t(l(t),t) - \bar{u}_t(l(t),t)), \tag{2.10}$$

which is a copy of the system (1.1)-(1.3) with the injection $a_1(u_t(l(t),t) - \bar{u}_t(l(t),t))$ and the constant a_1 is to be determined later. We define the disturbance estimate $\hat{d}(t)$ as

$$\hat{d}(t) = \bar{d}_x(0,t). \tag{2.11}$$

Remark 1. *The system* (2.5)-(2.7) *and* (2.8)-(2.10) *can be regarded as a disturbance estimator which uses available measurements* $u_t(l(t),t)$ $u(0,t)$ *to estimate the unknown disturbance in the original system (1.1)-(1.3).*

3 Convergence of Estimate Error

Theorem 3.1. *The error* $\tilde{d}(t)$ *between the disturbance estimate* $\hat{d}(t)$ *defined in* (2.11) *and the actual disturbance* $d(t)$ *is exponentially convergent in the sense of the following equation:*

$$|\tilde{d}(t)| = |d(t) - \hat{d}(t)| \le V_{\tilde{d}0}e^{-\lambda_2 t}, \forall t \ge 0,$$

where $\lambda_2 > 0$ *and* $V_{\tilde{d}0}$ *is a positive constant which depends on the initial value only.*

In order to prove Theorem 3.1, we define $\check{u}(x,t) = u(x,t) - \bar{u}(x,t)$ as

$$\check{u}_{tt}(x,t) = \check{u}_{xx}(x,t), \tag{3.12}$$

$$\check{u}_x(0,t) = d(t), \tag{3.13}$$

$$\check{u}_x(l(t),t) = -a_1\check{u}_t(l(t),t), \tag{3.14}$$

and $\tilde{v}(x,t) = \check{u}(x,t) - \bar{d}(x,t)$ as

$$\tilde{v}_{tt}(x,t) = \tilde{v}_{xx}(x,t), \tag{3.15}$$

$$\tilde{v}(0,t) = 0, \tag{3.16}$$

$$\tilde{v}_x(l(t),t) = -a_1\tilde{v}_t(l(t),t). \tag{3.17}$$

We present two lemmas first.

Lemma 3.1. *For any initial data* $(\tilde{v}(x,0), \tilde{v}_t(x,0))$ *which belong to* $H^1(0,L) \times L^2(0,L)$, *the system* (3.15)-(3.17) *is exponentially stable in the sense of the norm*

$$\left(\int_0^{l(t)} \tilde{v}_t^2(x,t)dx + \int_0^{l(t)} \tilde{v}_x^2(x,t)dx\right)^{1/2}.$$

Proof. The following system norm will be used:

$$\Omega_{\tilde{v}}(t) = \|\tilde{v}_t(\cdot,t)\|^2 + \|\tilde{v}_x(\cdot,t)\|^2, \tag{3.18}$$

where $\|\tilde{v}(\cdot,t)\|^2$ is a compact notation for $\int_0^{l(t)} \tilde{v}(x,t)^2 dx$. In addition, we employ a Lyapunov function

$$V_{\tilde{v}}(t) = \frac{1}{2}\|\tilde{v}_t(\cdot,t)\|^2 + \frac{1}{2}\|\tilde{v}_x(\cdot,t)\|^2 + \delta_{\tilde{v}}\int_0^{l(t)} (1+x)\tilde{v}_x(x,t)\tilde{v}_t(x,t)dx, \tag{3.19}$$

where the parameter $\delta_{\tilde{v}}$ is to be determined and needs to at least satisfy

$$0 < \delta_{\tilde{v}} < \frac{1}{1+L} \tag{3.20}$$

to guarantee the positive definiteness of $V_{\tilde{v}}(t)$. Then we can get the following inequality:

$$\theta_{\tilde{v}1}\Omega_{\tilde{v}}(t) \le V_{\tilde{v}}(t) \le \theta_{\tilde{v}2}\Omega_{\tilde{v}}(t), \tag{3.21}$$

where

$$\theta_{\tilde{v}1} = \frac{1}{2} - \frac{1}{2}\delta_{\tilde{v}}(1+L), \tag{3.22}$$

$$\theta_{\tilde{v}2} = \frac{1}{2} + \frac{1}{2}\delta_{\tilde{v}}(1+L). \tag{3.23}$$

Taking the derivative of $V_{\tilde{v}}(t)$ along the system (3.15)-(3.17), we obtain

$$\begin{aligned}\dot{V}_{\tilde{v}} = &-a_1\tilde{v}_t^2(l(t),t)\\ &- \frac{1}{2}\left|\dot{l}(t)\right|\tilde{v}_t^2(l(t),t) - a_1^2\frac{1}{2}\left|\dot{l}(t)\right|\tilde{v}_t^2(l(t),t)\\ &+ \frac{\delta_{\tilde{v}}}{2}(1+l(t))\tilde{v}_t^2(l(t),t)\\ &+ {a_1}^2\frac{\delta_{\tilde{v}}}{2}(1+l(t))\tilde{v}_t^2(l(t),t) - \frac{\delta_{\tilde{v}}}{2}\tilde{v}_x^2(0,t)\\ &- \frac{\delta_{\tilde{v}}}{2}\|\tilde{v}_t\|^2 - \frac{\delta_{\tilde{v}}}{2}\|\tilde{v}_x\|^2\\ &+ \left|\dot{l}(t)\right| a_1\delta_{\tilde{v}}(1+l(t))\tilde{v}_t^2(l(t),t)\end{aligned}$$

$$\leq -\frac{\delta_{\tilde v}}{2}\|\tilde v_t\|^2 - \frac{\delta_{\tilde v}}{2}\|\tilde v_x\|^2$$
$$- \left(a_1 - \frac{\delta_{\tilde v}(1+L)}{2}(1+{a_1}^2)\right)\tilde v_t^2(l(t),t)$$
$$- \left|\dot l(t)\right|\left(\frac{1}{2} + \frac{{a_1}^2}{2} - a_1\delta_{\tilde v}(1+L)\right)\tilde v_t^2(l(t),t)$$
$$- \frac{\delta_{\tilde v}}{2}\tilde v_x^2(0,t)$$
$$\leq -\lambda_1 V_{\tilde v}, \tag{3.24}$$

where

$$\lambda_1 = \frac{\delta_{\tilde v}}{2\theta_{\tilde v 2}}, \tag{3.25}$$

and $\delta_{\tilde v}$ is chosen to satisfy

$$0 < \delta_{\tilde v} < \frac{1}{1+L}\min\left\{1, \frac{2a_1}{1+{a_1}^2}, \frac{1+{a_1}^2}{2a_1}\right\} \tag{3.26}$$

to make sure the coefficients before the term $\tilde v_t^2(l(t),t)$ are negative.

Lemma 3.2. *For any initial data* $(e(x,0), e_t(x,0))$ *which belong to* $H^1(0,L)\times L^2(0,L)$, *the system* $e(x,t) = \tilde v_t(x,t)$ *is exponentially stable such that*

$$|e(x,t)| \leq V_{e0}e^{-\lambda_2 t}, \forall t \geq 0,$$

where $\lambda_2 > 0$ *and* V_{e0} *is a positive constant which only depends on the initial values. Then we obtain*

$$|\tilde v_x(0,t)| \leq V_{\tilde v 0}e^{-\lambda_2 t}, \forall t \geq 0,$$

where $V_{\tilde v 0}$ *is a positive constant which only depends on the initial values.*

Proof. According to the system (3.15)-(3.17), the e system can be written as

$$e_{tt}(x,t) = e_{xx}(x,t), \tag{3.27}$$
$$e(0,t) = 0, \tag{3.28}$$
$$e_x(l(t),t) = -b_1 e_t(l(t),t), \tag{3.29}$$

where

$$b_1 = \frac{a_1 - |\dot l(t)|}{1 - a_1|\dot l(t)|}. \tag{3.30}$$

We can choose

$$|\dot l(t)| < a_1 < \frac{1}{|\dot l(t)|} \tag{3.31}$$

in the case of $|\dot l(t)| \leq 1$, and chose

$$\frac{1}{|\dot l(t)|} < a_1 < |\dot l(t)|, \tag{3.32}$$

when $|\dot l(t)| > 1$, to make

$$b_1 > 0. \tag{3.33}$$

Consider a Lyapunov function for the system (3.27)-(3.29),

$$V_e(t) = \frac{1}{2}\|e_t(\cdot,t)\|^2 + \frac{1}{2}\|e_x(\cdot,t)\|^2$$
$$+ \delta_e\int_0^{l(t)}(1+x)e_x(x,t)e_t(x,t)dx, \tag{3.34}$$

where the parameter δ_e is to be determined and needs to at least satisfy

$$0 < \delta_e < \frac{1}{1+L} \tag{3.35}$$

to guarantee the positive definiteness of $V_e(t)$.

Taking the derivative of $V_e(t)$ along the system (3.27)-(3.29), through a similar computation as (3.24), we get the exponential stability of the system $e(x,t)$,

$$\dot V_e \leq -\frac{\delta_e}{2}\|e_t\|^2 - \frac{\delta_e}{2}\|e_x\|^2$$
$$- \left(b_1 - \frac{\delta_e(1+L)}{2}(1+{b_1}^2)\right){e_t}^2(l(t),t)$$
$$- \left|\dot l(t)\right|\left(\frac{1}{2} + \frac{{b_1}^2}{2} - b_1\delta_e(1+L)\right)e_t^2(l(t),t)$$
$$- \frac{\delta_e}{2}{e_x}^2(0,t)$$
$$\leq -\lambda_2 V_e, \tag{3.36}$$

where

$$\lambda_2 = \frac{\delta_e}{2\mu_e}, \tag{3.37}$$

and δ_e and μ_e satisfy:

$$0 < \delta_e < \frac{1}{1+L}\min\left\{1, \frac{2b_1}{1+{b_1}^2}, \frac{1+{b_1}^2}{2b_1}\right\}, \tag{3.38}$$

$$\mu_e = \frac{1}{2} + \frac{1}{2}\delta_e(1+L). \tag{3.39}$$

According to $\tilde v$ system (3.15)-(3.17), from Cauchy-Schwarz inequality, we obtain

$$|\tilde v_x(0,t)| \leq |\tilde v_x(l(t),t)| + \left|\int_0^{l(t)}\tilde v_{xx}(x,t)dx\right|$$
$$\leq |a_1\tilde v_t(l(t),t)| + L\left(\int_0^{l(t)}|\tilde v_{tt}(x,t)|^2dx\right)^{\frac{1}{2}}$$
$$= |a_1 e(l(t),t)| + L\|e_t(\cdot,t)\|$$
$$\leq a_1|\tilde v_t(L,0)|e^{-\lambda_2 t} + L\|\tilde v_{tt}(\cdot,0)\|e^{-\lambda_2 t}, \tag{3.40}$$

then

$$|\tilde{v}_x(0,t)| \le \left(a_1|\tilde{v}_t(L,0)| + L\,\|\tilde{v}_{tt}(\cdot,0)\|\right)e^{-\lambda_2 t}. \tag{3.41}$$

The proof is thus completed.

With Lemma 3.2, we can then prove Theorem 3.1.

Proof. According to (2.11) and (3.13), the estimated error of the proposed disturbance estimator can be obtained as

$$\begin{aligned}\tilde{d}(t) &= d(t) - \hat{d}(t)\\ &= \check{u}_x(0,t) - \bar{d}_x(0,t)\\ &= \tilde{v}_x(0,t).\end{aligned} \tag{3.42}$$

According to Lemma 3.2, we can conclude Theorem 3.1.

4 Boundedness of the Observer State.

THEOREM 4.1. *All states in the disturbance observer are bounded in the sense of,*

$$\begin{aligned}\sup_{t\ge 0}\Bigg[\int_0^{l(t)} &(\bar{d}_t^2(x,t) + \bar{d}_x^2(x,t)\\ &+ \bar{u}_t^2(x,t) + \bar{u}_x^2(x,t))dx\Bigg] < \infty.\end{aligned} \tag{4.43}$$

In order to prove Theorem 4.1, we propose one Lemma first.

LEMMA 4.1. *The system* (3.12)-(3.14) *is uniformly bounded in the sense of*

$$\sup_{t\ge 0}\left[\int_0^{l(t)} (\check{u}_t^2(x,t) + \check{u}_x^2(x,t))\right] < \infty. \tag{4.44}$$

Proof. The following system norm will be used:

$$\Omega_{\check{u}}(t) = \|\check{u}_t(\cdot,t)\|^2 + \|\check{u}_x(\cdot,t)\|^2. \tag{4.45}$$

In addition, we employ a Lyapunov function

$$\begin{aligned}V_{\check{u}}(t) &= \frac{1}{2}\|\check{u}_t(\cdot,t)\|^2 + \frac{1}{2}\|\check{u}_x(\cdot,t)\|^2\\ &+ \delta_{\check{u}}\int_0^{l(t)}(1+x)\check{u}_x(x,t)\check{u}_t(x,t)dx,\end{aligned} \tag{4.46}$$

where the parameter $\delta_{\check{u}}$ is to be determined and needs to at least satisfy

$$0 < \delta_{\check{u}} < \frac{1}{1+L} \tag{4.47}$$

to guarantee the positive definiteness of $V_{\check{u}}(t)$. Then we can get the following inequality:

$$\theta_{\check{u}1}\Omega_{\check{u}}(t) \le V_{\check{u}}(t) \le \theta_{\check{u}2}\Omega_{\check{u}}(t), \tag{4.48}$$

where

$$\theta_{\check{u}1} = \frac{1}{2} - \frac{1}{2}\delta_{\check{u}}(1+L), \tag{4.49}$$

$$\theta_{\check{u}2} = \frac{1}{2} + \frac{1}{2}\delta_{\check{u}}(1+L). \tag{4.50}$$

Taking the derivative of $V_{\check{u}}(t)$ along the system (3.12)-(3.14), we obtain

$$\begin{aligned}\dot{V}_{\check{u}} &= -a_1\check{u}_t^2(l(t),t) - \check{u}_t(0,t)\check{u}_x(0,t)\\ &- \frac{1}{2}\left|\dot{l}(t)\right|\check{u}_t^2(l(t),t) - a_1^2\frac{1}{2}\left|\dot{l}(t)\right|\check{u}_t^2(l(t),t)\\ &+ \frac{\delta_{\check{u}}}{2}(1+l(t))\check{u}_t^2(l(t),t) - \frac{\delta_{\check{u}}}{2}\check{u}_t^2(0,t)\\ &+ {a_1}^2\frac{\delta_{\check{u}}}{2}(1+l(t))\check{u}_t^2(l(t),t) - \frac{\delta_{\check{u}}}{2}\check{u}_x^2(0,t)\\ &- \frac{\delta_{\check{u}}}{2}\|\check{u}_t\|^2 - \frac{\delta_{\check{u}}}{2}q\|\check{u}_x\|^2\\ &+ \left|\dot{l}(t)\right|a_1\delta_{\check{u}}(1+l(t))\check{u}_t^2(l(t),t).\end{aligned}$$

Applying Young's inequality,

$$\begin{aligned}\dot{V}_{\check{u}} &\le -\frac{\delta_{\check{u}}}{2}\|\check{u}_t\|^2 - \frac{\delta_{\check{u}}}{2}\|\check{u}_x\|^2\\ &- \left(a_1 - \frac{\delta_{\check{u}}(1+L)}{2}(1+{a_1}^2)\right)\check{u}_t^2(l(t),t)\\ &- \left|\dot{l}(t)\right|\left(\frac{1}{2} + \frac{{a_1}^2}{2} - a_1\delta_{\check{u}}(1+L)\right)\check{u}_t^2(l(t),t)\\ &- (\frac{\delta_{\check{u}}}{2} - \frac{r_1}{2})\check{u}_t^2(0,t) + (\frac{1}{2r_1} - \frac{\delta_{\check{u}}}{2})\check{u}_x^2(0,t).\end{aligned}$$

By choosing

$$0 < \delta_{\check{u}} < \frac{1}{1+L}\min\left\{1, \frac{2a_1}{1+{a_1}^2}, \frac{1+{a_1}^2}{2a_1}\right\}, \tag{4.51}$$

$$0 < r_1 < \delta_{\check{u}}, \tag{4.52}$$

we obtain,

$$\dot{V}_{\check{u}} \le -\lambda_0 V_{\check{u}} + \overline{M}, \tag{4.53}$$

where,

$$\lambda_0 = \frac{\delta_{\check{u}}}{2\theta_{\check{u}2}}, \tag{4.54}$$

and $\overline{M} = (\frac{1}{2r_1} - \frac{\delta_{\check{u}}}{2})\overline{D}^2$.

Multiplying both sides of (4.53) by $e^{\lambda_0 t}$, we obtain

$$\frac{d(V_{\check{u}}e^{\lambda_0 t})}{dt} \le \overline{M}e^{\lambda_0 t}. \tag{4.55}$$

Integration of (4.55) yields

(4.56)

$$\Omega_{\check{u}}(t) \le \frac{1}{\theta_{\check{u}1}}V_{\check{u}}(t) \le \frac{1}{\theta_{\check{u}1}}(V_{\check{u}}(0) - \frac{\overline{M}}{\lambda_0})e^{-\lambda_0 t} + \frac{\overline{M}}{\theta_{\check{u}1}\lambda_0},$$

which implies $\Omega_{\check{u}}(t)$ is uniformly bounded by $\frac{1}{\theta_{\check{u}1}} V_{\check{u}}(0)$. Moreover, it is uniformly ultimately bounded with the ultimate bound $\frac{\overline{M}}{\theta_{\check{u}1}\lambda_0}$.

According to Lemma 4.1, we can prove Theorem 4.1 now.

Proof. We can design the control input $U(t)$ to ensure $u(x,t)$ uniformly bounded. Based on Lemma 4.1 which prove the uniform boundedness of the system $\check{u}(x,t)$ and Lemma 3.1 which means the exponential stability of $\tilde{v}(x,t)$ system, we can get $\bar{d}(x,t)$ is uniformly bounded considering $\tilde{v}(x,t) = \check{u}(x,t) - \bar{d}(x,t)$. Then we can obtain that $\bar{u}(x,t)$ is also uniformly bounded considering $\tilde{v}(x,t) = u(x,t) - \bar{u}(x,t) - \bar{d}(x,t)$. Therefore, all states in the observer system (2.5)-(2.10) are uniformly bounded. The proof is completed.

5 Conclusion

In this paper, we propose a disturbance observer for wave PDEs on a time-varying domain, subject to unknown anti-collocated disturbances. This method can be applied in ADRC where the estimation of disturbance is required and used to attenuate the actual disturbance by feedback control in wave PDEs. Exponential convergence of the estimate error and uniform boundedness of all states in the proposed observer have been proved by Lyapunov analysis. Physically the observer system can be used in cable elevators to estimate the disturbance at the cage.

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