

Enhanced Battery State Estimation: Electrolyte Lithium-ion Concentration Observer

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Abstract

Closed-loop Partial Differential Equation (PDE) observers for electrolyte lithium concentration are designed providing exact estimations of this parameter within the negative and positive electrodes and the separator.

Keywords

Li-ion battery; State estimation; Electrolyte concentration, Backstepping observer.

I. Introduction

In lithium-ion batteries, the concentration of lithium ions in the electrolyte phase is one of the key electrochemical states. This state is considered among the most significant information in lithium-ion batteries. To monitor this parameter, PDE backstepping observers for electrolyte lithium concentration $c_e^\pm(t, 0^\pm)$ are designed, which are provably convergent state observers. Modified boundary conditions of electrolyte lithium concentration are employed for observer design.

II. Problem Statement

Battery Model: Single Particle Model with electrolyte (SPMe) dynamics

Input: Current of battery $I(t)$

Output: Terminal voltage of battery $V(t)$

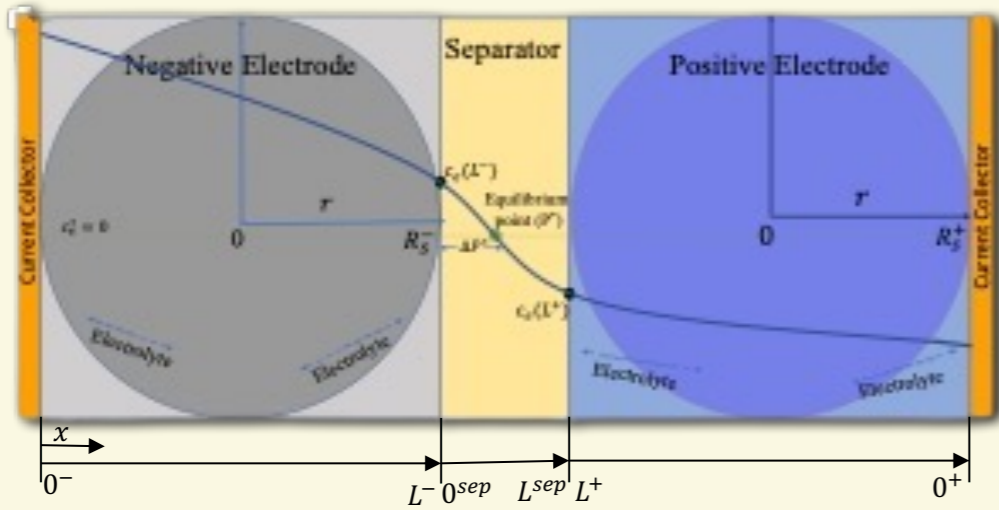


Fig. 1. SPMc Schematic with equilibrium point for modified boundary conditions.

The backstepping observer is designed using the electrolyte lithium concentration error at the battery terminals $c_e^\pm(t, 0^\pm) - \hat{c}_e^\pm(t, 0^\pm)$. To obtain $c_e^\pm(t, 0^\pm)$, the measurable battery parameters related to electrolyte lithium concentration, such as electrolyte resistance and diffusion coefficient, will first be measured using the EIS technique. Then, employing these measurements, $c_e^\pm(t, 0^\pm)$ will be obtained through the Einstein relationship for injection into the observer.

$$\text{Einstein relationship: } c_e^\pm(t, 0^\pm) = \frac{k_B T \kappa_e^\pm(c_e^\pm(t, 0^\pm))}{e^2 N_A D_e^\pm(t, 0^\pm)}$$

Liquid-phase Lithium concentration:

$$\frac{\partial c_e^-(t, x)}{\partial t} = \frac{\partial}{\partial x} \left[\frac{D_e^-, \text{eff}(c_e^-)}{\varepsilon_e^-} \frac{\partial c_e^-(t, x)}{\partial x} \right] + \frac{(1-t_c^0)}{\varepsilon_e^- F L^-} I(t),$$

$$\frac{\partial c_e^{\text{sep}}(t, x)}{\partial t} = \frac{\partial}{\partial x} \left[\frac{D_e^{\text{sep}, \text{eff}}(c_e^{\text{sep}})}{\varepsilon_e^{\text{sep}}} \frac{\partial c_e^{\text{sep}}(t, x)}{\partial x} \right],$$

$$\frac{\partial c_e^+(t, x)}{\partial t} = \frac{\partial}{\partial x} \left[\frac{D_e^+, \text{eff}(c_e^+)}{\varepsilon_e^+} \frac{\partial c_e^+(t, x)}{\partial x} \right] - \frac{(1-t_c^0)}{\varepsilon_e^+ F L^+} I(t).$$

Observer for the electrolyte lithium concentration

In the negative electrode

$$\frac{\partial \hat{c}_e^-(t, x)}{\partial t} = \frac{\partial^2 \hat{c}_e^-(t, x)}{\partial x^2} + \frac{L^-(1-t_c^0)}{D_e^-, \text{eff} F} I(t) + p_1(x)(c_e^-(t, 0^-) - \hat{c}_e^-(t, 0^-)),$$

$$\frac{\partial \hat{c}_e^-(t, 0^-)}{\partial x} = p_{10}(c_e^-(t, 0^-) - \hat{c}_e^-(t, 0^-)),$$

$$\frac{\partial \hat{c}_e^-(t, 1^-)}{\partial x} = \frac{-L^- \hat{c}_e^-(t, 1^-)}{\left(\frac{L^{\text{sep}} + L^+ - L^-}{2} \right)},$$

In the positive electrode:

$$\frac{\partial \hat{c}_e^+(t, x)}{\partial t} = \frac{\partial^2 \hat{c}_e^+(t, x)}{\partial x^2} - \frac{L^+(1-t_c^0)}{D_e^+, \text{eff} F} I(t) + p_1(x)(c_e^+(t, 0^+) - \hat{c}_e^+(t, 0^+)),$$

$$\frac{\partial \hat{c}_e^+(t, 0^+)}{\partial x} = p_{10}(c_e^+(t, 0^+) - \hat{c}_e^+(t, 0^+)),$$

$$\frac{\partial \hat{c}_e^+(t, 1^+)}{\partial x} = \frac{-D_e^-, \text{eff} \hat{c}_e^+(t, 1^+)}{L^+ \left(\frac{L^{\text{sep}} + L^+ - L^-}{2} \right)}.$$

In the separator:

$$\frac{\partial \hat{c}_e^{\text{sep}}(t, x)}{\partial t} = \frac{D_e^{\text{sep}, \text{eff}}}{\varepsilon_e^{\text{sep}}} \frac{\partial^2 \hat{c}_e^{\text{sep}}(t, x)}{\partial x^2} + p_1(x)(c_e^{\text{sep}}(t, 0^{\text{sep}}) - \hat{c}_e^{\text{sep}}(t, 0^{\text{sep}})),$$

$$\frac{\partial \hat{c}_e^{\text{sep}}(t, 0^{\text{sep}})}{\partial x} = \frac{\hat{c}_e^-(t, 0^{\text{sep}})}{\frac{L^{\text{sep}} + L^+ - L^-}{2}} - p_{10} \hat{c}_e^{\text{sep}}(t, 0^{\text{sep}}),$$

$$\frac{\partial \hat{c}_e^{\text{sep}}(t, L^{\text{sep}})}{\partial x} = \frac{\hat{c}_e^+(t, L^{\text{sep}})}{\frac{L^{\text{sep}} - L^+ + L^-}{2}}.$$

Observer gains: $p_1(x) = \alpha \bar{p}_y(x, 0)$,

$$p_{10} = -\frac{\alpha}{2},$$

where

$$p(x, y) = -\alpha(1-y) \frac{I_1 \left(\sqrt{\alpha} \left((1-y)^2 - (1-x)^2 \right) \right)}{\sqrt{\alpha} \left((1-y)^2 - (1-x)^2 \right)}.$$

III. Simulation and Contributions

- On a LiFePO₄ battery cell
- Cell capacity of 2.3 Ah.
- Input: Urban Dynamometer Driving Schedule (UDDS) current profile
- Temperature: constant at T = 298 K.

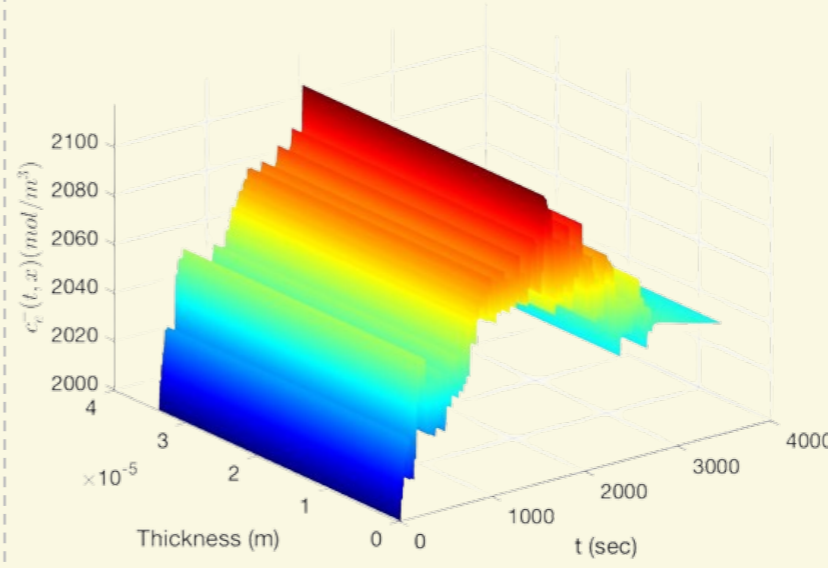


Fig 2. Electrolyte lithium concentration in the negative electrode

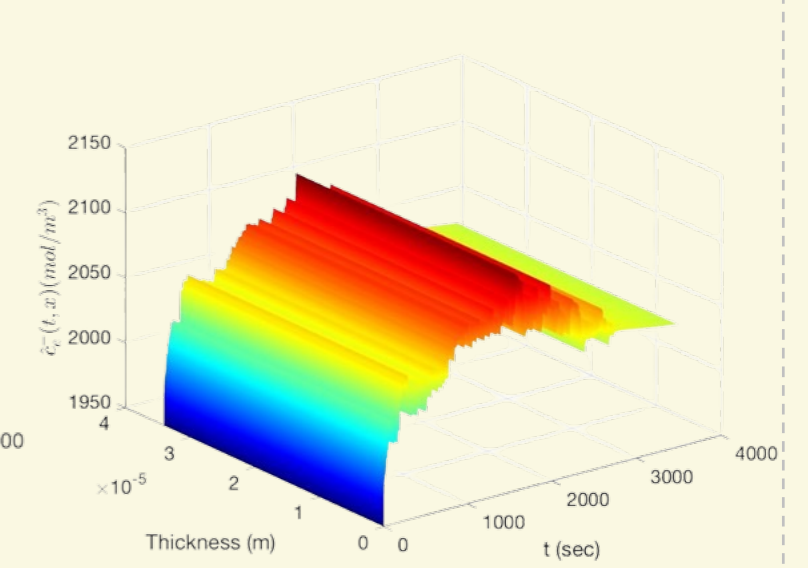


Fig 3. Estimation of electrolyte lithium concentration in the negative electrode

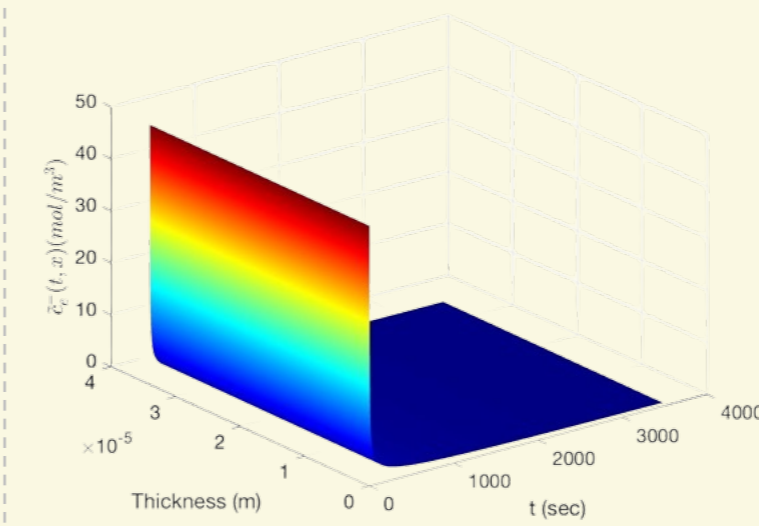


Fig 4. Estimation error of electrolyte lithium concentration in the negative electrode

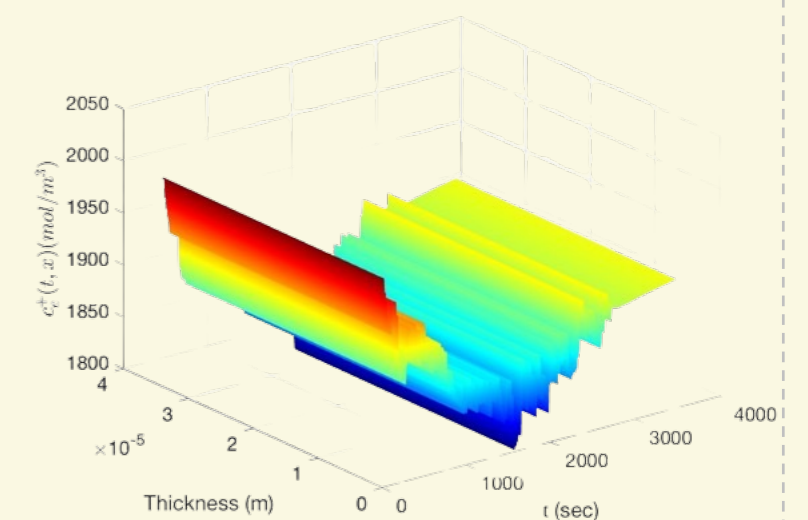


Fig 5. Electrolyte lithium concentration in the positive electrode

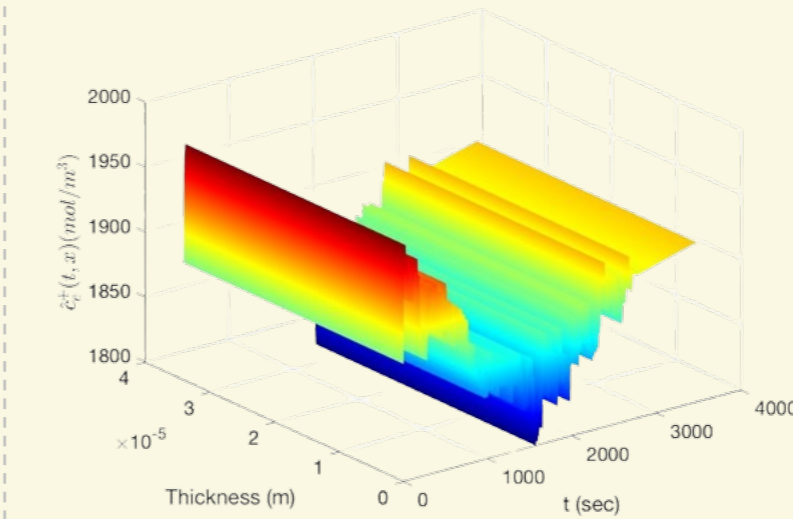


Fig 6. Estimation of electrolyte lithium concentration in the positive electrode

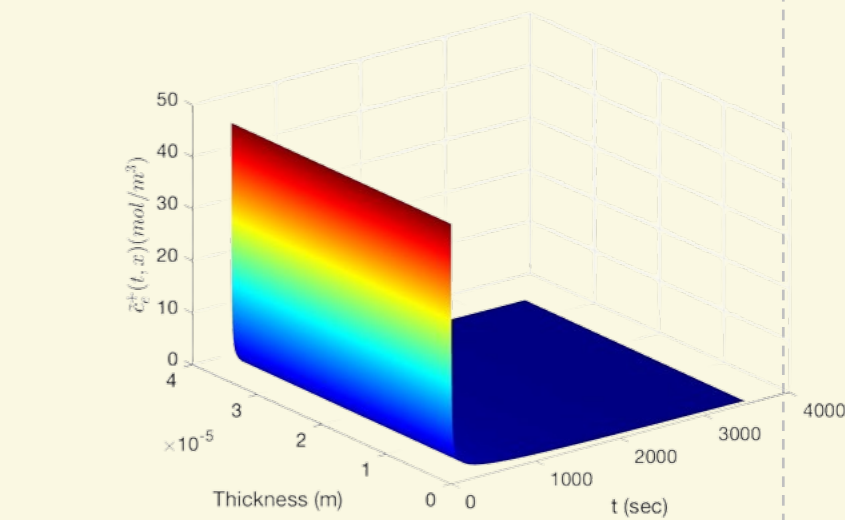


Fig 7. Estimation error of electrolyte lithium concentration in the positive electrode

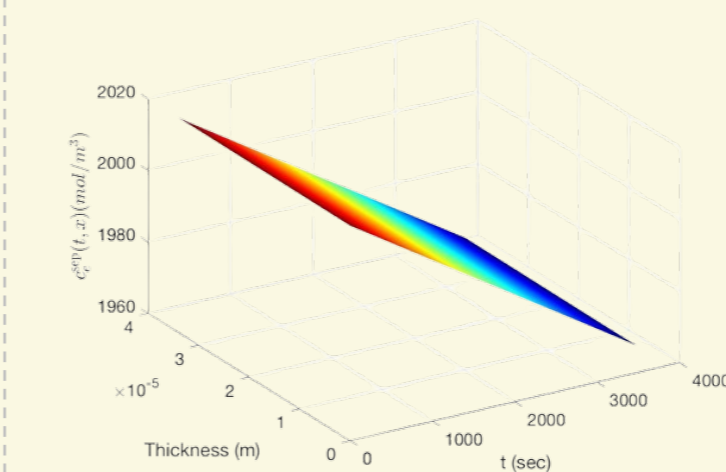


Fig 8. Electrolyte lithium concentration in the separator

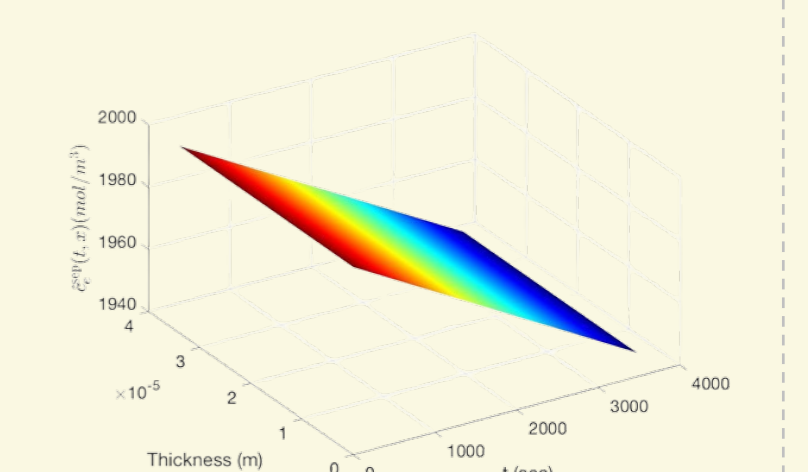


Fig 9. Estimation of electrolyte lithium concentration in the separator

Linear trends in the separator region can be observed in Fig. 8 and 9, perfectly aligning with the quasi-linear assumption used to derive the modified boundary condition.

Contributions:

- To the best of the author's knowledge, this is the first time closed-loop PDE observers are designed for electrolyte lithium concentration in lithium-ion batteries.
- in this study SPMc is employed, representing a more accurate battery model compared to SPM.
- Discretization of the lithium concentration PDEs in the model is avoided.

VI. Conclusion and Future works

This paper proposes a provably convergent PDE backstepping state observer for the exact estimation of electrolyte lithium concentration in different regions of lithium-ion batteries.

Future works:

- Using the original boundary conditions instead of modified ones
- Considering temperature and aging effects
- Integration of event-triggered scheme into the observer.

Reference:

Sara Sepasiahooyi and Shu-Xia Tang, Enhanced Battery State Estimation: Part 1, IEEE Conference on Decision and Control, 2024, Under review.