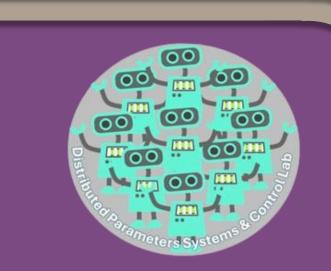
# Temperature Estimation in Lithium-ion Batteries Through Cascaded Electrochemical-Thermal Models



Patryck Ferreira and Shu-Xia Tang E-mails: patferre@ttu.edu and shuxia.tang@ttu.edu



### Introduction

- Lithium-ion batteries rely on either liquid or solid electrolytes for ion transport.
- Modeling batteries is crucial in preventing thermal runaway and ensuring safety.
- Estimators play a key role in enhancing the performance and safety of the battery by predicting unmeasured variables.
- Multiple Partial Differential Equations (PDEs) are employed to describe the dynamics of lithium-ion concentration. As an example, the following PDE is used to model the concentration of lithium ions in the electrolyte of All-Solid-State Batteries (ASSB).

$$\frac{\partial c_{\rm e}}{\partial t}(y,t) = \frac{2D_{\rm Li^+}D_{\rm n^-}}{D_{\rm Li^+} + D_{\rm n^-}} \frac{\partial^2 c_{\rm e}}{\partial y^2}(y,t) + r(y,t),$$

$$c_{\rm e}(y,0) = \delta c_{\rm e,0},$$

$$\frac{\partial c_{\rm e}(0,t)}{\partial y} = -\frac{I(t)}{2FAD_{\rm Li^+}},$$

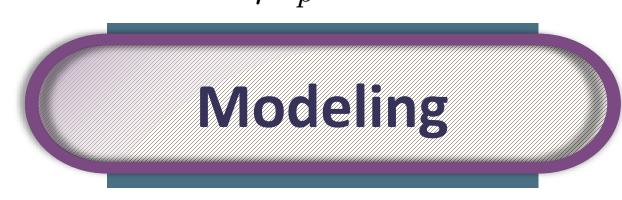
$$\frac{\partial c_{\rm e}(L,t)}{\partial y} = -\frac{I(t)}{2FAD_{\rm Li^+}},$$

- The voltage V(t) for (ASSB) is given by:  $V(t) = E_{eq}(\bar{\theta}_s(t)) + \eta_t(t).$
- Electrochemical heat:

$$S(t) = V(t)I(t).$$

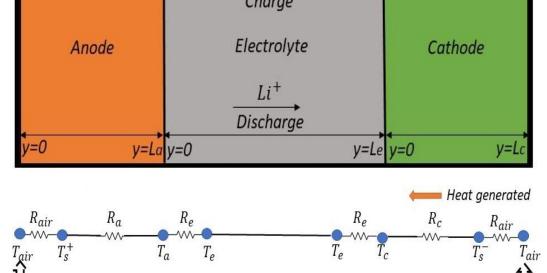
Temperature dynamics:

$$\dot{T}(t) = \frac{S(t) + q_{\text{cond}}(t) + q_{\text{conv}}(t)}{\rho c_p v}$$

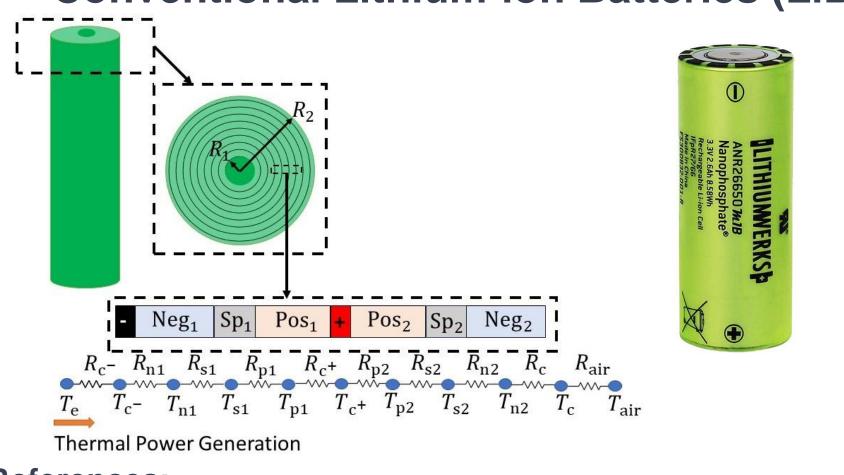


All-Solid-State Batteries (ASSB)





#### Conventional Lithium-ion Batteries (LiBs)



#### All-Solid-State Batteries (ASSB)

$$\dot{\mathbf{T}}(t) = A\mathbf{T}(t) + B\mathbf{u}(t) + \mathbf{w}(t),$$

$$y(t) = C\mathbf{T}(t) + v(t),$$

$$\mathbf{w} \sim (0, Q),$$

$$v \sim (0, R),$$

$$A = \begin{bmatrix} \frac{-R_{\rm c} - R_{\rm air}}{\lambda_{\rm air} R_{\rm c} R_{\rm air}} & \frac{1}{\lambda_{\rm air} R_{\rm c}} & 0 & 0 & 0\\ \frac{1}{\lambda_{\rm c} R_{\rm c}} & \frac{-1}{\lambda_{\rm c}} & 0 & 0 & 0\\ 0 & \frac{1}{\lambda_{\rm e} R_{\rm e}} & \frac{-1}{\lambda_{\rm e} R_{\rm e}} & 0 & 0\\ 0 & 0 & \frac{1}{\lambda_{\rm a} R_{\rm e}} & \frac{-1}{\lambda_{\rm a} R_{\rm e}} & 0\\ 0 & 0 & 0 & \frac{1}{\lambda_{\rm air} R_{\rm a}} & \frac{R_{\rm a} - R_{\rm air}}{\lambda_{\rm air} R_{\rm a} R_{\rm air}} \end{bmatrix}$$

$$\mathbf{T}(t) = \begin{bmatrix} T_{s}^{-}(t) & T_{c}(t) & T_{e}(t) & T_{a}(t) & T_{s}^{+}(t) \end{bmatrix}^{\text{tr}},$$

$$B = \begin{bmatrix} \frac{1}{\lambda_{\text{air}}R_{\text{air}}} & 0 & 0 & 0 & \frac{-1}{\lambda_{\text{air}}R_{\text{air}}} \\ 0 & \frac{1}{\lambda_{c}} & \frac{1}{\lambda_{e}} & \frac{1}{\lambda_{a}} & 0 \end{bmatrix}^{\text{tr}},$$

$$\mathbf{u}(t) = \begin{bmatrix} T_{\text{air}}(t) & S(t) \end{bmatrix}^{\text{tr}},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

#### Conventional Lithium-ion Batteries (LiBs)

$$\dot{\mathbf{T}}(t) = A\mathbf{T}(t) + B\mathbf{u}(t),$$
  
 $\mathbf{y}(t) = C\mathbf{T}(t),$ 

where A is a 10x10 matrix, B is a 10x2 matrix, **T** is a 1x10 state vector, and **u** is a 2x1 input vector, C is explored in the simulation results.



#### Kalman filter.

Input: Initialize the state estimate:  $\hat{T}(0)$ ; Initialize the error covariance matrix: P(0); Set the process noise covariance: Q; Set the measurement noise covariance: R; Output: Temperature Estimation:  $\hat{T}$ ;

for k=1 to length(t) do

Initialization Equation:  $\hat{T}(0)=E[T(0)]$ Initialization of Error Covariance Matrix:  $P(0)=E\left[(T(0)-\hat{T}(0))(T(0)-\hat{T}(0))^{tr}\right]$ Kalman Gain Equation:  $K=PC^{tr}R^{-1}$ State Estimation Update Equation:  $\hat{T}=A\hat{T}+Bu+K(y-C\hat{T})$ Error Covariance Matrix Update Equation:  $\hat{P}=-PC^{tr}R^{-1}CP+AP+PA^{tr}+Q$ 

Luenberger observer.

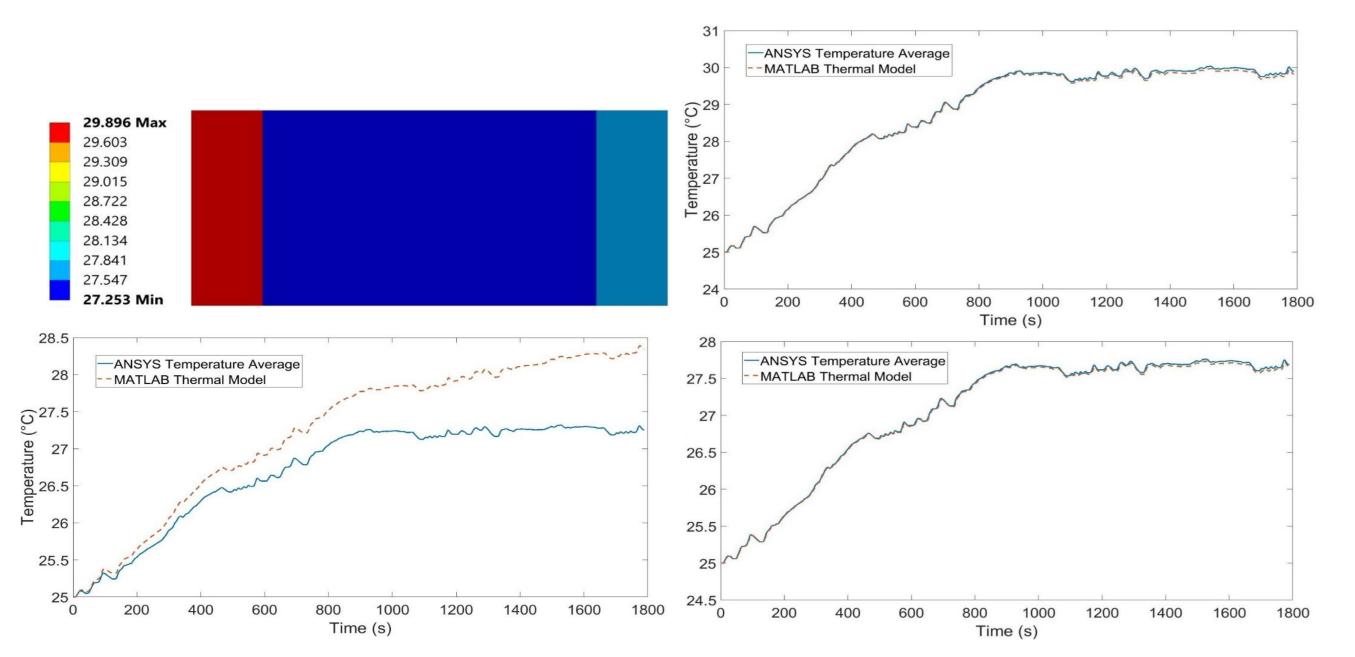
$$\hat{T}(t) = A\hat{T}(t) + Bu(t) + L[CT(t) - C\hat{T}(t)],$$

• Error dynamics:  $\dot{\tilde{T}}(t) = (A - LC)\tilde{T}(t)$ .

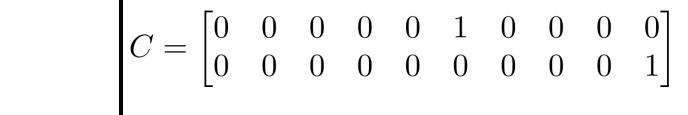
• Lyapunov function 
$$V(\tilde{T}(t)) = \tilde{T}(t)^{tr} P \tilde{T}(t),$$

## Verification and Simulation

Verification of the ASSB thermal model.



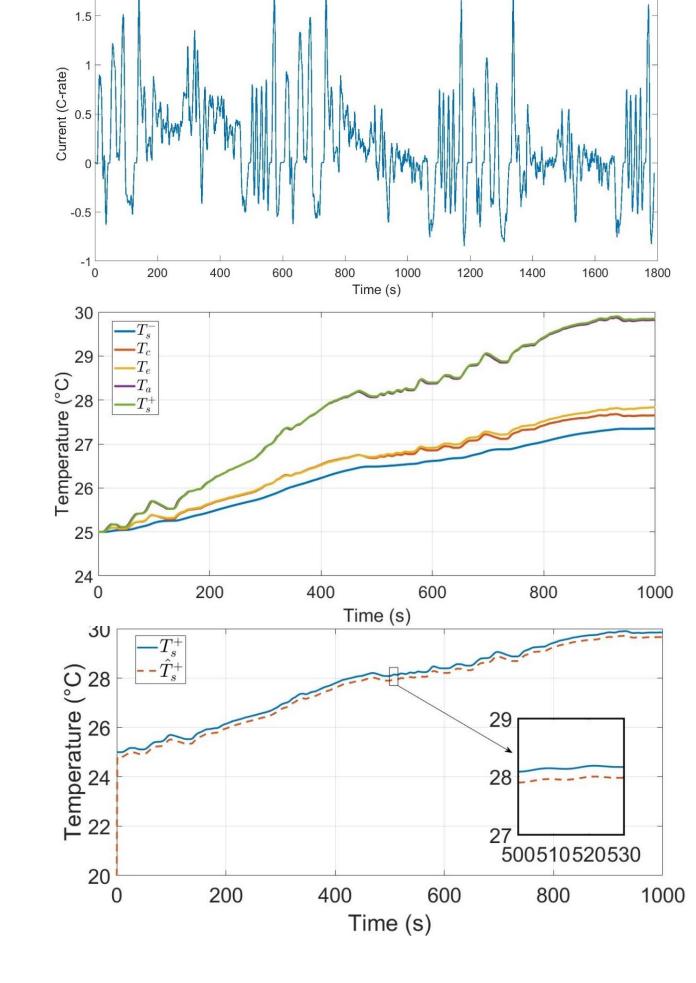


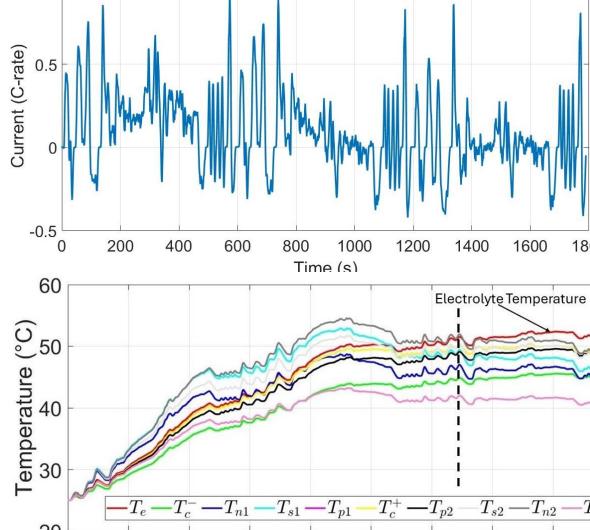


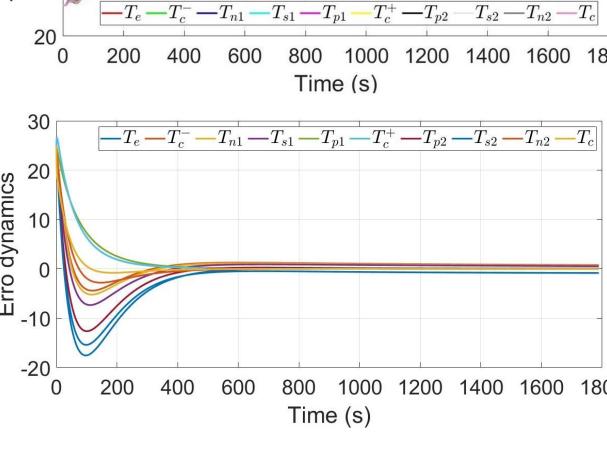
LiBs Simulation Results:

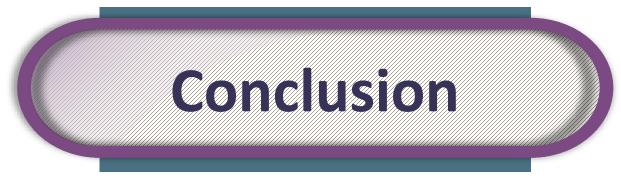
LiBs optimal sensor placement:











- A thermal model, constructed using the energy balance approach, was validated through Ansys transient thermal simulations, showing minimal differences.
- A Kalman filter for ASSB aligned well with the true model, and error dynamics in the Luenberger Observer for lithium-ion batteries (LIBs) approached zero.
- Future work includes relaxing the adiabatic assumption and experimental validation.

#### References: