

Answer to Most Frequent Questions

Please email your questions and suggestions to shuxiatang.control@gmail.com.

1. **Question:** Derivation of the inequality on the 11th line of the right column of Page 542, which is the same as the inequality (78) of

Susto and Krstic, "Control of PDE-ODE cascades with Neumann interconnections", Journal of the Franklin Institute, 2010,

and also the same as the 3rd last line of Page 2148 of

Tang and Xie, "Stabilization for a coupled PDE-ODE control system", Journal of the Franklin Institute, 2011.

Answer: We have

$$-\frac{1}{2}\|w_x\|^2 - \frac{1}{2}\|w_{xx}\|^2 \leq \int_0^x w_x w_{xx} dx = \frac{1}{2}w_x(x)^2 - \frac{1}{2}w_x(0)^2, \quad (1)$$

where the LHS uses the Young's inequality, and the RHS is the result from direct calculation. Taking the integrals of both sides, we obtain

$$-l\|w_x\|^2 - l\|w_{xx}\|^2 \leq \|w_x\|^2 - lw_x(0)^2, \quad (2)$$

that is,

$$-l\|w_{xx}\|^2 \leq (1+l)\|w_x\|^2 - lw_x(0, t)^2. \quad (3)$$

The following inequality then follows trivially:

$$-\|w_{xx}\|^2 \leq \frac{1+l}{l}\|w_x\|^2 - w_x(0, t)^2. \quad (4)$$