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# Observer-based Stabilization of Stochastic Hamiltonian Systems

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***Abstract*— This paper propose an observer-based stabilization method for stochastic Hamiltonian systems. First, for stochastic Hamiltonian systems without parameter uncertainty, we construct a state observer and design an observer-based stabilization controller such that the closed loop system is asymptotically stable. Then, we put forward an adaptive observer and a stabilization controller for stochastic nonlinear Hamiltonian systems with parameter uncertainty. The asymptotical convergence of the observers is shown without constructing the estimation error system and the Lyapunov functions are constructed by the Hamiltonian function. The internal structure of the system is fully utilized during the observer design and stability analysis. A numerical example is given to illustrate the effectiveness of the proposed method.**

## I. INTRODUCTION

Observer design is of great importance for many dynamical systems because the full state measurements of the systems are generally not available and thus the full state feedback method are not applicable to these systems [1]- [6]. Recently, the observer design and observer-based control of stochastic nonlinear system gained more and more attention because the stochastic model can describe more accurately the dynamics of the practical engineering systems subjected to stochastic noises and disturbances [7]. In [8], Barbata et al addressed the robust observer design of nonlinear stochastic systems with unstructured and norm-bounded parameter uncertainties to guarantee the estimation error almost surely exponential stability. They also investigated the robust reduced order observer design of stochastic nonlinear [9]. Observer-based stabilization of stochastic nonlinear systems was addressed in [10] and [11]. For the uncertain stochastic nonlinear systems with time-delay and actuator nonlinearities, Yin et al [12] proposed an observed-based $H_\infty$ controller. [13] concerned the observer based stochastic trajectory tracking control of mechanical systems under the assumption that only the position measurements can be available.

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It is well-known that for the observer-based control of nonlinear systems, it is generally difficult to find a suitable Lyapunov function to verify the asymptotical convergence of the observer error systems and the asymptotical stability of the closed loop systems. In this paper, we propose an observer-based stabilization method for stochastic Hamiltonian systems where the Hamiltonian function and internal structure are utilized to construct the Lyapunov functions and to complete the stability analysis of the closed system. It should be pointed out that stochastic Hamiltonian systems are of great importance because many physical systems, such as power systems, mechanical systems, possess an internal energy transformation, energy dissipation property and can be re-formulated as Hamiltonian systems [14]- [16]. The observer-based control for two kinds of stochastic Hamiltonian systems is addressed, i.e., the systems without parameter uncertainty and the systems in presence of parameter uncertainty. First, for stochastic Hamiltonian systems without parameter uncertainty, we construct a state observer and design a feedback controller such that the closed loop system is asymptotically stable and the states of the estimation system asymptotically converges to those of the considered stochastic Hamiltonian systems. Then, for stochastic Hamiltonian systems in presence of parameter uncertainty, we put forward an adaptive observer and a stabilization controller based on the estimated state. The asymptotically convergence of the observer state is shown without constructing an estimation error system and the Lyapunov function is constructed by the Hamiltonian function of the system. Numerical Example demonstrates the effectiveness of the proposed method.

The rest of the paper is organized as follows. The observer-based stabilization of stochastic Hamiltonian systems without parameter uncertainty is considered in Section 2. In Section 3, the adaptive observer-based design problem is addressed. In Section 4, we give an example to demonstrate the effectiveness of the proposed method. Finally, Section 5 summarizes the paper and draws the conclusion.

## II. OBSERVER-BASED STABILIZATION OF STOCHASTIC HAMILTONIAN SYSTEMS

Consider the following stochastic Hamiltonian systems [17] [18]

$$\Sigma : \begin{cases} dx = \left[ \big(J(x) - R(x)\big) \dfrac{\partial H(x)}{\partial x} + g(x)u \right] dt \\ \qquad + g_w(x)dw, \\ y = g^T(x) \dfrac{\partial H(x)}{\partial x}, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t)$, $y(t) \in \mathbb{R}^m$ are the state, the control input and the output of the system respectively.

$w(t) \in \mathbb{R}^r$ is the standard Wiener process defined on a probability space$(\Omega, \mathcal{F}, \mathcal{P})$ with $\Omega$ being a sample space, $\mathcal{F}$ being the sigma algebra of the observable random events and $\mathcal{P}$ being probability measure on $\Omega$. $J(x) \in \mathbb{R}^{n\times n}$ is skew-symmetric and $R(x) \in \mathbb{R}^{n\times n}$ is positive semi-definite. $H(x)$ is generally regarded as the Hamiltonian function of the system and achieves minimum at $x_0$. Without loss of generality, we suppose $x_0$ is an equilibrium point of the system.

*Assumption 2.1:* The Hamiltonian function $H(x) > 0$ for all $x \neq 0$ and the stochastic Hamiltonian system (1) is dissipative with the supply rate $y^T u$, that is, the following inequality holds

$$\begin{aligned} &-\frac{\partial^T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}(x)^T \frac{\partial^2 H(x)}{\partial x} \tilde{g}(x) \right\} \leq 0. \end{aligned} \tag{2}$$

Moreover, there exist a matrix $D(x) \leq 0$, such that

$$\begin{aligned} &-\frac{\partial^T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}(x)^T \frac{\partial^2 H(x)}{\partial x} \tilde{g}(x) \right\} \\ &= -\frac{\partial^T H(x)}{\partial x} D(x) \frac{\partial H(x)}{\partial x}. \end{aligned} \tag{3}$$

*Assumption 2.2:* $\dfrac{\partial H(x)}{\partial x} \neq 0 (x \neq 0)$ and the system is zero state detectable with respect to the virtual output $y_v = D^{\frac{1}{2}} \dfrac{\partial H(x)}{\partial x}$ and input variable $u$.

*Assumption 2.3:* There exist non-zero matrices $K(x)$, $K_1(x)$, and semi-positive definite matrices $W(x)$ and $W_1(x)$ with

$$W(x) = R(x) + [g(x)K(x) + K^T(x)g^T(x)] \geq 0, \tag{4}$$

$$\begin{aligned} &-\frac{\partial^T H(x)}{\partial x} W(x) \frac{\partial H(x)}{\partial x} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{ \tilde{g}(x)^T \frac{\partial^2 H(x)}{\partial x} \tilde{g}(x) \right\} \\ &= -\frac{\partial^T H(x)}{\partial x} W_1(x) \frac{\partial H(x)}{\partial x} \leq 0, \end{aligned} \tag{5}$$

$$K(x) = K_1(x) W_1(x), \tag{6}$$

and the system

$$dx = [J(x) - W(x)] \frac{\partial H(x)}{\partial x} dt \tag{7}$$

is zero state detectable with respect to $y_w = W_1^{\frac{1}{2}}(x) \dfrac{\partial H(x)}{\partial x}$.

For the stochastic Hamiltonian system (1), consider the following state observer

$$\begin{aligned} \hat{\Sigma} : d\hat{x} = &\left[J(\hat{x}) - R(\hat{x})\right] \frac{\partial H(\hat{x})}{\partial \hat{x}} dt + g(\hat{x}) u dt \\ &+ g_w(\hat{x}) dw + K^T(\hat{x}) \left[y - g^T(\hat{x}) \frac{\partial H(\hat{x})}{\partial x}\right] dt. \end{aligned} \tag{8}$$

Rewrite system (1) and (8) in a compact form

$$dX = \left[\bar{J}(X) - \bar{R}(X)\right] \frac{\partial \bar{H}}{\partial X} dt + \bar{g}(X) u dt + \bar{g}_w(X) dw \tag{9}$$

where $X = [x^T, \hat{x}^T]^T$, $\bar{H}(X) = H(x) + H(\hat{x})$,

$$\frac{\partial \hat{H}(X)}{\partial X} = \begin{bmatrix} \dfrac{\partial H(x)}{\partial x} \\ \dfrac{\partial H(\hat{x})}{\partial \hat{x}} \end{bmatrix},$$

$$\bar{J}(X) = \begin{bmatrix} J(x) & 0 \\ 0 & J(\hat{x}) \end{bmatrix},$$

$$\bar{R}(X) = \begin{bmatrix} R(x) & 0 \\ -K^T(\hat{x}) g^T(x) & R(\hat{x}) + K^T(\hat{x}) g^T(\hat{x}) \end{bmatrix},$$

$$\bar{g}(X) = \begin{bmatrix} g(x) \\ \bar{g}(\hat{x}) \end{bmatrix}, \quad \bar{g}_w(X) = \begin{bmatrix} g_w(x) \\ \bar{g}_w(\hat{x}) \end{bmatrix}.$$

Noticing that $K(\hat{x}) \neq 0, g(x) \neq 0$, $\bar{R}(X)$ is not symmetric, so we need to construct a feedback controller to make the system a standard stochastic Hamiltonian system formulation. Choosing the feedback controller as follows

$$u = -K(\hat{x}) \frac{\partial H(\hat{x})}{\partial \hat{x}} + v, \tag{10}$$

where $v$ is a virtual input.

The closed loop system can be written as

$$dX = \left[\tilde{J}(X) - \tilde{R}(X)\right] \frac{\partial \bar{H}}{\partial X} dt + \bar{g}(X) v dt + \bar{g}_w(X) dw, \tag{11}$$

where

$$\tilde{J}(X) = \begin{bmatrix} J(x) & -g(x)K(\hat{x}) \\ K^T(\hat{x}) g^T(x) & J(\hat{x}) \end{bmatrix},$$

$$\tilde{R}(X) = \begin{bmatrix} R(x) & 0 \\ 0 & R(\hat{x}) + g(\hat{x})K(\hat{x}) + K^T(\hat{x}) g^T(\hat{x}) \end{bmatrix}.$$

*Theorem 2.1:* Suppose Assumptions 2.1 - 2.3 hold. The stochastic Hamiltonian system (1) can be stabilized by the observer-based feedback controller (10) and (8) is an observer of the system.

**Proof**: Along the trajectories of the system (9) in absence of the control input $v$, we have

$$\begin{aligned} \mathcal{L}\bar{H}(X) \quad = \quad & -\frac{\partial^T H(x)}{\partial x} R(x) \frac{\partial H(x)}{\partial x} \\ & + \frac{1}{2}\mathrm{Tr}\left\{ g_w(x)^T \frac{\partial^2 H(x)}{\partial x} g_w(x) \right\} \\ & - \frac{\partial^T H(\hat{x})}{\partial \hat{x}} W(\hat{x}) \frac{\partial H(\hat{x})}{\partial \hat{x}} \\ & + \frac{1}{2}\mathrm{Tr}\left\{ g_w(\hat{x})^T \frac{\partial^2 H(\hat{x})}{\partial \hat{x}} g_w(\hat{x}) \right\} \\ \leq \quad & -\frac{\partial^T H(\hat{x})}{\partial \hat{x}} D(x) \frac{\partial H(\hat{x})}{\partial \hat{x}} \\ & - \frac{\partial^T H(\hat{x})}{\partial \hat{x}} W_1(x) \frac{\partial H(\hat{x})}{\partial \hat{x}} \\ \leq \quad & 0 \end{aligned} \tag{12}$$

So the closed loop system in absence of the control input $v$, i.e., $v = 0$ is stable in probability. Moreover, the trajectories of the system (11) converge to the following largest invariant set

$$\begin{aligned} S = \left\{ X | \mathcal{L}\bar{H}(X) = 0 \right\} = \Bigg\{ & X | D^{\frac{1}{2}}(x) \frac{\partial H(x)}{\partial x} = 0, \\ & W_1^{\frac{1}{2}}(\hat{x}) \frac{\partial H(\hat{x})}{\partial \hat{x}} = 0 \Bigg\}. \end{aligned} \tag{13}$$

Taking into consideration of the Assumption 2.1 and Assumption 2.3, $W_1^{\frac{1}{2}}(\hat{x})\frac{\partial H(\hat{x})}{\partial \hat{x}} = 0$ indicates $K(\hat{x})\frac{\partial H(\hat{x})}{\partial \hat{x}} = 0$. So when $v = 0$ and $W_1^{\frac{1}{2}}(\hat{x})\frac{\partial H(\hat{x})}{\partial \hat{x}} = 0$, (11) can be equivalently re-written as

$$\begin{cases} dx = \left[\big(J(x) - R(x)\big)\frac{\partial H(x)}{\partial x}\right] dt + g_w(x)dw, \\ d\hat{x} = \big[J(\hat{x}) - W(\hat{x})\big]\frac{\partial H(\hat{x})}{\partial \hat{x}} dt + g_w(\hat{x})dw \\ \qquad + K^T(\hat{x})y^T(x)\frac{\partial H(x)}{\partial x}\big] dt \end{cases} \tag{14}$$

Further noticing that system (1) is dissipative and zero state detectable with respect to $y_1(x)$, we can see that first subsystem of (14) is asymptotically stable in probability. Moreover, from $y_1(x) \to 0$ and thus $x \to x_0$, we have $\frac{\partial H(x)}{\partial x} \to 0$. So the second subsystem of (14) can be written as

$$d\hat{x} = \big[J(\hat{x}) - W(\hat{x})\big]\frac{\partial H(\hat{x})}{\partial \hat{x}} dt + g_w(\hat{x})dw \tag{15}$$

It is obvious that the system is a dissipative Hamiltonian system and the infinitesimal operator $\mathcal{L}\hat{H}(\hat{x}) = -\frac{\partial^T H(\hat{x})}{\partial \hat{x}} W_1(x)\frac{\partial H(\hat{x})}{\partial \hat{x}} \leq 0$. From the zero state detectability with respect to $y_2$, we get $\hat{x} \to 0, t \to \infty$ in probability from $y_2 = W_1^{\frac{1}{2}}(\hat{x})\frac{\partial H(\hat{x})}{\partial \hat{x}} = 0$. So the system (11) has only one point $(x_0^T, x_0^T)^T$ in the largest invariant set $S$. From the LaSalle's invariant principle of stochastic nonlinear systems, we can see that the closed loop system is asymptotically stable in probability and $||x - \hat{x}|| \leq ||x|| + ||\hat{x}|| \to 0$ in probability as $t \to \infty$. □

*Remark 2.1:* The feedback stabilization controller (10) is constructed from the state $\hat{x}$ and is realizable. The configuration of the observer system is shown in Fig. 1.

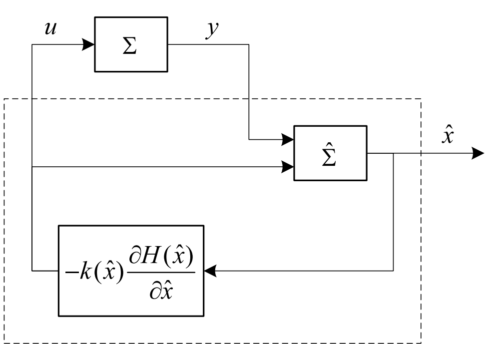


Fig. 1: Configuration of the observer-based stabilization controller

## III. Adaptive Observer-based Stabilization of Stochastic Hamiltonian Systems

In this section, we consider the observer design of the following stochastic Hamiltonian systems subjected to parameter perturbations

$$\Sigma_p : \begin{cases} dx = \left[\big(J(x,p) - R(x,p)\big)\frac{\partial H(x,p)}{\partial x}\right] dt \\ \qquad + g(x)udt + g_w(x)dw, \\ y = g^T(x)\frac{\partial H(x,0)}{\partial x}, \end{cases} \tag{16}$$

where $p$ is the bounded uncertain parameter perturbation.

The objective of this section is to seek an adaptive observer in the form of

$$\begin{cases} d\hat{x} = \alpha(\hat{x}, \hat{\theta}, y, u), \\ \dot{\hat{\theta}} = \beta(\hat{x}, \hat{\theta}, y, u), \end{cases} \tag{17}$$

such that $||x - \hat{x}|| \to (t \to \infty)$ in probability, where $\hat{\theta}$ is the estimator of $\theta$ and $\theta$ is the unknown vector related with $p$ and an observer-based stabilization controller such that the closed loop system is asymptotically stable in probability.

*Assumption 3.1:* The Hamiltonian function $H(x, 0) > 0$ for all $x \neq 0$ and the stochastic system (16) is strict dissipative with respect to $H(x,p)$, that is, the following inequality holds

$$\begin{aligned} &-\frac{\partial^T H(x,p)}{\partial x} R(x,p)\frac{\partial H(x,p)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{\tilde{g}(x)^T\frac{\partial^2 H(x,p)}{\partial x}\tilde{g}(x)\right\} \leq 0. \end{aligned} \tag{18}$$

Moreover, there exist a positive definite matrix $D(x,p)$ such that

$$\begin{aligned} &-\frac{\partial^T H(x,p)}{\partial x} R(x,p)\frac{\partial H(x,p)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{\tilde{g}(x)^T\frac{\partial^2 H(x,p)}{\partial x}\tilde{g}(x)\right\} \\ &= -\frac{\partial^T H(x,p)}{\partial x} D(x,p)\frac{\partial H(x,p)}{\partial x}. \end{aligned} \tag{19}$$

*Assumption 3.2:* There exist non-zero matrices $K_1(x)$, $K_1(x)$ and positive definite matrices $W(x)$ and $W_1(x,p)$ with

$$W(x) = R(x,0) + [g(x)K(x) + K^T(x)g^T(x)] > 0, \tag{20}$$

$$\begin{aligned} &-\frac{\partial^T H(x,p)}{\partial x} W(x)\frac{\partial H(x,p)}{\partial x} \\ &+\frac{1}{2}\mathrm{Tr}\left\{\tilde{g}(x)^T\frac{\partial^2 H(x,p)}{\partial x}\tilde{g}(x)\right\} \\ &= -\frac{\partial^T H(x,p)}{\partial x} W_1(x,p)\frac{\partial H(x,p)}{\partial x} < 0. \end{aligned} \tag{21}$$

*Assumption 3.3:* There exist a proper constant matrix $\Theta$ such that

$$[J(x,p) - R(x,p)] \triangle_H(x,p) = g(x)\Theta^T\theta, \tag{22}$$

where $\triangle_H(x,p) = \frac{\partial H(x,p)}{\partial x} - \frac{\partial H(x,0)}{\partial x}$.

According to (22), (16) can be written as follows

$$\begin{cases} dx = \left[\big(J(x,\theta) - R(x,\theta)\big)\frac{\partial H(x,0)}{\partial x} + g(x)\Theta^T\theta\right] dt \\ \qquad + g(x)udt + g_w(x)dw, \\ y = g^T(x)\frac{\partial H(x,0)}{\partial x}, \end{cases} \tag{23}$$

Suppose the structure of the system can not be duplicated. Consider the following adaptive observer

$$\hat{\Sigma}_p : \begin{cases} d\hat{x} = \left[\big(J(\hat{x},0) - R(\hat{x},0)\big)\dfrac{\partial H(\hat{x},0)}{\partial \hat{x}} + g(\hat{x})\Theta^T\theta\right]dt \\ \quad +K^T(\hat{x})\left[y - g^T(\hat{x})\dfrac{\partial H(\hat{x},0)}{\partial \hat{x}}\right]g(\hat{x})udt \\ \quad +g_w(\hat{x})dw, \\ \dot{\hat{\theta}} = Q\Theta y, \end{cases} \tag{24}$$

where $Q > 0$ is the adaptive gain.

Rewriting (23) and (24) in a compact form, we have

$$\begin{cases} \begin{bmatrix} dx \\ d\hat{x} \end{bmatrix} = \begin{bmatrix} J(x,p) - R(x,p) & 0 \\ K^T(\hat{x})g^T(x) & \Phi(\hat{x}) \end{bmatrix} \\ \quad \times \begin{bmatrix} \dfrac{\partial H(x,0)}{\partial x} \\ \dfrac{\partial H(\hat{x},0}{\hat{x}} \end{bmatrix} dt + \begin{bmatrix} g(x)\Theta^T\theta \\ g(\hat{x})\Theta^T\hat{\theta} \end{bmatrix} dt \\ \quad + \begin{bmatrix} g(x) \\ g(\hat{x}) \end{bmatrix} udt + \begin{bmatrix} g_w(x) \\ g_w(\hat{x}) \end{bmatrix} dw, \\ \dot{\hat{\theta}} = Q\Theta g^T(x)\dfrac{\partial H(x,0)}{\partial x}. \end{cases} \tag{25}$$

where $\Phi(\hat{x}) = J(\hat{x},0) - R(\hat{x},0) - K^T(\hat{x})g^T(\hat{x})$.

In order to make (25) a dissipative stochastic Hamiltonian system, construct a feedback controller as follows

$$u = -K(\hat{x})\frac{\partial H(\hat{x},0)}{\partial \hat{x}} - \Theta^T\hat{\theta} + v, \tag{26}$$

where $v$ is the reference control input.

Substitute (26) into (25), we have

$$\begin{cases} \begin{bmatrix} dx \\ d\hat{x} \end{bmatrix} = \begin{bmatrix} J(x,p) - R(x,p) & -g(x)K(\hat{x}) \\ K^T(\hat{x})g^T(x) & \Phi(\hat{x}) \end{bmatrix} \\ \quad \times \begin{bmatrix} \dfrac{\partial H(x,0)}{\partial x} \\ \dfrac{\partial H(\hat{x},0}{\hat{x}} \end{bmatrix} dt \\ \quad + \begin{bmatrix} g(x) \\ g(\hat{x}) \end{bmatrix} vdt + \begin{bmatrix} g_w(x) \\ g_w(\hat{x}) \end{bmatrix} dw, \\ \dot{\hat{\theta}} = Q\Theta g^T(x)\dfrac{\partial H(x,0)}{\partial x}. \end{cases} \tag{27}$$

The above system can be equivalently written as

$$\begin{aligned} dX = \left[\bar{J}(x,p) - \bar{R}(x,p)\right]\frac{\partial H(X)}{\partial X}dt \\ +\bar{g}(X)vdt + \bar{g}_w(X)dw, \end{aligned} \tag{28}$$

where $X = [x^T, \hat{x}^T, \hat{\theta}^T]^T$,

$$\bar{H}(X) = H(x,0) + H(\hat{x},0) + \frac{1}{2}(\theta - \hat{\theta})^T Q^{-1}(\theta - \hat{\theta}),$$

$$\bar{J}(X,p) = \begin{bmatrix} J(x,p) & -g(x)K(\hat{x}) & -g(x)\Theta^T Q \\ K^T(\hat{x})g^T(x) & J(\hat{x},0) & 0 \\ Q\Theta g^T(x) & 0 & 0 \end{bmatrix},$$

$$\bar{R}(X,p) = \begin{bmatrix} R(x,p) & 0 & 0 \\ 0 & W(\hat{x}) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{H}(X) = \begin{bmatrix} \dfrac{\partial H(x,0)}{\partial x}, & \dfrac{\partial H(\hat{x},0)}{\partial \hat{x}}, & \dfrac{\partial \bar{H}(X)}{\partial \hat{\theta}} \end{bmatrix}^T,$$

$$\bar{g}(X) = \begin{bmatrix} g(x), & g(\hat{x}), & 0 \end{bmatrix}^T,$$

$$\bar{g}_w(X) = \begin{bmatrix} g_w(x), & g_w(\hat{x}), & 0 \end{bmatrix}^T.$$

From Assumptions 3.1 and 3.2, we can see that (28) is a dissipative stochastic Hamiltonian system.

*Theorem 3.1:* Suppose Assumptions 3.1 - 3.3 hold. The uncertain stochastic port-controlled Hamiltonian system (16) can be stabilized by the observer-based feedback controller (26) and (24) is an adaptive observer of the system .

**Proof**: According to the relationship between the dissipation and stability of stochastic nonlinear systems, we can see that the system (28) is stable in probability. Let $X_0 = [x_0^T, \hat{x}_0^T, \hat{\theta}_0^T]^T$ be the equilibrium point of the system. It is easy to get that $\dfrac{\partial H(x_0,0)}{\partial x} = \dfrac{\partial H(\hat{x}_0,0)}{\partial \hat{x}} = 0$. From Assumption 3.1, we have $\hat{x}_0 = x_0$.

Calculating the differential operator of $\bar{H}(X)$ along the trajectories of the system (28), we get

$$\begin{aligned} \mathcal{L}\bar{H}(X) = & -\frac{\partial^T \bar{H}(X)}{\partial X}\bar{R}(X,p)\frac{\partial \bar{H}(X)}{\partial X} \\ & +\frac{1}{2}\mathrm{Tr}\left\{\bar{g}_w(X)^T\frac{\partial^2 \bar{H}(X)}{\partial X}\bar{g}_w(X)\right\} \\ = & -\frac{\partial^T H(x,0)}{\partial x}R(x,p)\frac{\partial H(x,0)}{\partial x} \\ & -\frac{\partial^T H(\hat{x},0)}{\partial \hat{x}}\left[R(\hat{x},0) + \big(K^T(\hat{x})g^T(\hat{x})\right. \\ & \left. +g(\hat{x})K(\hat{x})\big)\right]\frac{\partial H(\hat{x},0)}{\partial \hat{x}} \\ & +\frac{1}{2}\mathrm{Tr}\left\{g_w(x)^T\frac{\partial^2 H(x,p)}{\partial x}g_w(x)\right\} \\ & +\frac{1}{2}\mathrm{Tr}\left\{g_w(\hat{x})^T\frac{\partial^2 H(\hat{x},p)}{\partial \hat{x}}g_w(\hat{x})\right\} \\ = & -\frac{\partial^T H(x,0)}{\partial x}D(x,p)\frac{\partial H(x,0)}{\partial x} \\ & -\frac{\partial^T H(\hat{x},0)}{\partial x}W_1(x,p)\frac{\partial H(\hat{x},0)}{\partial x} \\ < & \ 0 \end{aligned} \tag{29}$$

So the trajectories of the closed loop system converge in probability to the set

$$\begin{aligned} S = \left\{X|\mathcal{L}\bar{H}(X) = 0\right\} = \Bigg\{X|D(x,p)^{\frac{1}{2}}\frac{\partial H(x,0)}{\partial x} = 0, \\ W_1(x,p)^{\frac{1}{2}}(\hat{x})\frac{\partial H(\hat{x},0)}{\partial \hat{x}} = 0\Bigg\}. \end{aligned} \tag{30}$$

From $D(x,p)^{\frac{1}{2}}\dfrac{\partial H(x,0)}{\partial x} = 0$ we have $\dfrac{\partial H(x,0)}{\partial x} = 0$ and $x \to x_0, t \to \infty$ in probability from Assumption 3.1. Similarly we can get that $\hat{x} \to 0, t \to \infty$ in probability from $W_1^{\frac{1}{2}}(\hat{x})\dfrac{\partial H(\hat{x})}{\partial \hat{x}} = 0$. Furthermore, we have $||x - \hat{x}|| \leq ||x - x_0|| + ||\hat{x} - x_0|| \to 0$ in probability as $t \to \infty$.

*Remark 3.1:* The feedback stabilization controller (26) depends only on $\hat{x}$ and $\hat{\theta}$ and is realizable. The Configuration of the adaptive stabilization controller is indicated in Fig. 2.

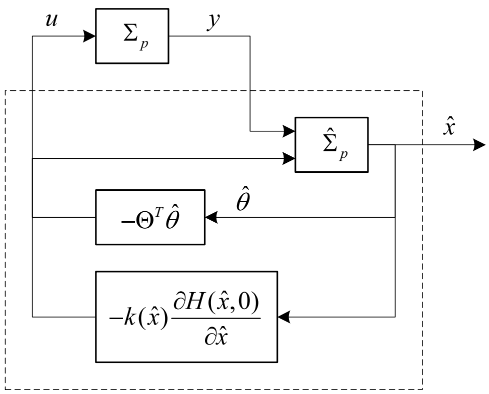


Fig. 2: Configuration of the observer-based adaptive stabilization controller

## IV. Numerical Example

Consider the adaptive observer-based stabilization of the uncertain stochastic Hamiltonian systems (16) with

$$J(x,p)=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$R(x,p)=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1+p \end{bmatrix},$$

$$g(x)=g_w(x)=\begin{bmatrix} 0, & 0, & x_3 \end{bmatrix}^T,$$

$H(x,p)=\frac{1}{2}ax_1^2+\frac{1}{2}bx_2^2+\frac{1}{2}(1+p)x_3^2$, $H(x,0)=H(x)$, $a>0$, $b>0$, $p$ is an unknown parameter satisfying $|p|<\frac{1}{4}$.

We verify that the Assumptions 3.1- 3.3 hold. First, it can be seen that the system is dissipative and there exist a matrix

$$D(x,p)=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \dfrac{p^2+2p+\frac{1}{4}}{(c+p)^2} \end{bmatrix}>0, \tag{31}$$

such that the Assumption 3.1 holds.

Second, let $K(x)=[0,0,x_3]\neq 0$, we have that

$$\begin{aligned} W(x)= & \; R(x,0)+\left[g(x)K(x)+K^T(x)g^T(x)\right] \\ = & \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c+2x_3^2 \end{bmatrix}>0, \end{aligned} \tag{32}$$

and that the positive matrix

$$W_1(x,p)=\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \dfrac{2(1+p)x_3^2+p+\frac{1}{2}}{c+p} \end{bmatrix} \tag{33}$$

satisfies the equation (21). Thus Assumption 3.2 holds.

Finally, direct calculation shows that

$$\triangle_H(x,p)=[0,0,px_3]^T. \tag{34}$$

Let $\theta=(c+p)p$, we can get

$$[J(x,p)-R(x,p)]\triangle_H(x,p)=g(x)\Theta^T\theta, \tag{35}$$

where $\Theta=1$. Thus Assumption 3.3 holds.

From Theorem 3.1, an adaptive observer for system (31) can be constructed as

$$\begin{cases} d\hat{x}=\left(\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}-\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\dfrac{\partial H(\hat{x})}{\partial \hat{x}}dt \\ \quad +\begin{bmatrix} 0 \\ 0 \\ \hat{x}_3 \end{bmatrix}\hat{\theta}dt+\begin{bmatrix} 0 \\ 0 \\ \hat{x}_3 \end{bmatrix}udt \\ \quad +\begin{bmatrix} 0 \\ 0 \\ \hat{x}_3 \end{bmatrix}(y-c\hat{x}_3^2)dt+\begin{bmatrix} 0 \\ 0 \\ \hat{x}_3 \end{bmatrix}dw, \\ \dot{\hat{\theta}}=Qg^T(x)\dfrac{\partial H(x)}{\partial x}, \end{cases} \tag{36}$$

where $Q>0$ is a constant. The observer-based stabilization controller can be constructed as

$$u=-K(\hat{x})\frac{\partial H(\hat{x})}{\partial \hat{x}}-\Theta^T\hat{\theta}=-c\hat{x}_3^2-\hat{\theta}. \tag{37}$$

Simulation results are shown in Fig. 3 $\sim$ Fig. 6. From the simulation results, we can see that the observer states can converge to the system states and the proposed adaptive observer-based stabilization controller can asymptotically stabilize the considered uncertain stochastic Hamiltonian system effectively.

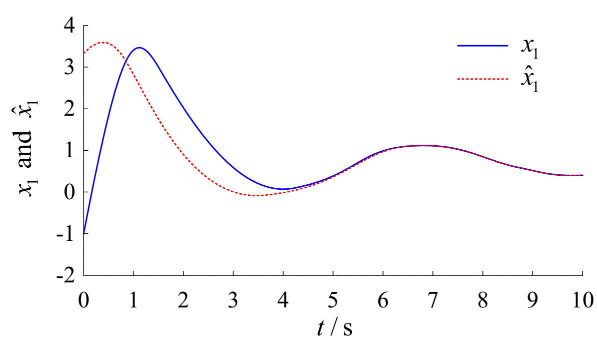


Fig. 3: the response of the system state and the observer state

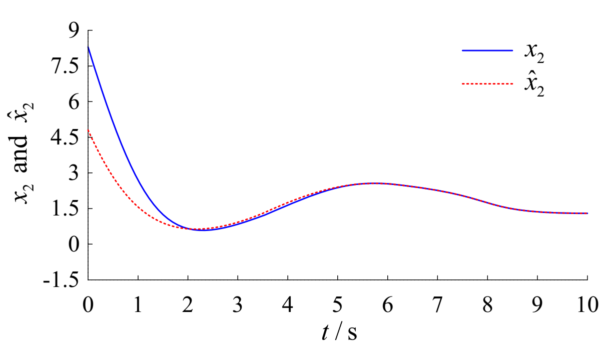


Fig. 4: the response of the system state and observer state

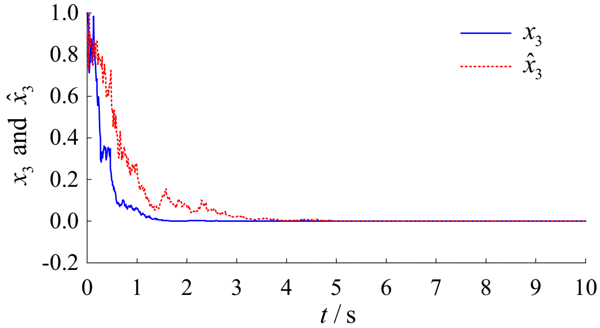


Fig. 5: the response of the system state and observer state

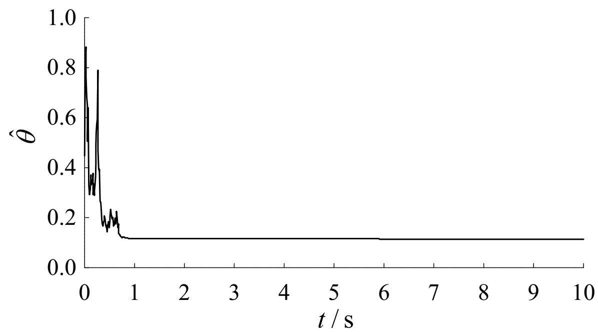


Fig. 6: the response of the estimated parameter

## V. Conclusion

The paper investigate the observer-based control of the stochastic Hamiltonian systems without parameter uncertainty and the systems in presence of parameter uncertainty. First, for stochastic Hamiltonian systems without parameter uncertainty, we put forward a state observer and propose an observer-based feedback controller to make the closed loop system asymptotically stable and the states of the estimation system asymptotically convergent. Then, for stochastic Hamiltonian systems subjected to parameter uncertainty, we put forward an adaptive observer and a stabilization controller based on the estimated state by utilizing the internal structure and dissipation property. The asymptotically convergence of the observer state is shown without constructing an estimation error system and the Lyapunov function for the closed loop system can be constructed by the Hamiltonian function. Numerical Example illustrates the effectiveness of the proposed method.

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