2019 American Control Conference (ACC)
Philadelphia, PA, USA, July 10-12, 2019

Backstepping-Based Time-Gap Regulation for Platoons

Fang-Chieh Chou, Shu-Xia Tang, Xiao-Yun Lu and Alexandre Bayen

Abstract— **The time-gap regulation problem for a cascaded system consisting of platooned automated vehicles following a leading non-automated vehicle is investigated in this article. Under the assumption of uniform boundedness of the acceleration of the leading vehicle, a control design scheme is proposed via an extension of integral backstepping control method, where additional terms that counter the impact due to the speed change of the non-automated vehicle are used. Each automated vehicle is actuated by one backstepping controller, demonstrated by a recursive control design procedure based on induction. As a result, both the time-gap error and the speed error between each pair of consecutive vehicles are proven to be ultimately bounded by some constants that can be tuned to be arbitrarily close to zero. In particular, the regulated time-gap guarantees enough time for the following vehicle in each pair to react to the velocity change of its preceding vehicle. Simulation is carried out to validate the proposed controllers.**

Index Terms— **Platoon, time-gap regulation, backstepping, induction.**

I. Introduction

Background. In late 1980' and early 1990', California PATH demonstrated the concept of *automated highway systems* (AHS) [1].However, most of the vehicles driving on the road are still driven by humans, and the same will likely hold in the foreseeable future. With the rise of automation, traffic will consist of mixed autonomy for years to come. Designing controllers that considers the impact of human-driven vehicles is thus an indispensable ingredient in the traffic management. In this article, we propose a controller for automated platoons considering the impact of human-driven vehicle driving in front of the platoon.

Automated platoon is a group of automated vehicles driving closely where vehicles among platoons are connected via wireless communication. It can potentially improve traffic mobility, fuel efficiency and driving safety. Overviews of the vehicular platoon control can be found in [2] [3]. Gap regulation controllers are essential to guarantee enough gap for the following vehicle in each pair of consecutive vehicles to react to the speed change of its preceding vehicle. A variety of gap policies are available in the existing literature, where the constant distance-gap policy [4] and constant time-gap policy [5] are most frequently adopted. Note that the constant time-gap policy is a policy that sets separation distance equal to vehicle speed times the desired time-gap constant, which is more reasonable than the constant distance-gap policy for two reasons. Firstly, it is more perceptually comfortable for human drivers because it mimics human driving behavior. Secondly, it is safer because long following distance allows more reaction time at high speed. Gap regulation control can be autonomous or cooperative. The autonomous approach relies on on-board sensors to detect the preceding vehicle. Many related works can be found in [6]. Cooperative approach needs a group of automated vehicles forming a platoon and uses wireless communication to exchange information. This article focuses on the cooperative approach.

Vehicles in the platoon can be potentially benefited by sharing information with each other. There are different ways of exploiting information of automated vehicles in the platoon. One way of controller design is sharing the information of the first vehicle with each following vehicle in the platoon, where control input is computed based on the states of the first vehicle and its preceding vehicle [7] [8] [9]. In another work, instead of using the information of the first vehicle, the states of two preceding vehicles ahead are used for the controller design [10]. Wu et al. [11] proposed a platoon control using consensus control method. Zheng et al. [12] proposed a model predictive control based approach for platoon control. More literature review with regard to control designs of automated vehicle platoons can be found in [13].

The integral backstepping approach allows systematic controller design for nonlinear *ordinary differential equation* (ODE) systems in strict feedback form [14]. Perry et al. [15] used backstepping for automated highway system platoon for reference speed tracking but not for car following controller. Wei et al. adopted backstepping controller for designing distance-gap regulation controller [4], but not for time-gap regulation, which is preferred for highway driving due to safety concern. One of the difficulties of the backstepping approach is that the virtual input becomes more complex as more stages are included during the design. Swaroop et al. [16] proposed a dynamic surface controller design to reduce the complexity of controller design.

Main Contributions. The present article considers a platoon of automated vehicles driving behind a non-automated vehicle. The main contributions of the work include the following:

- The time-gaps between each pair of consecutive vehicles are regulated by a designed controller based on the backstepping method;
- The gap error and the speed error between each pair of consecutive vehicles are proven to be ultimately

F.-C. Chou is with Department of Mechanical Engineering, UC Berkeley, Berkeley, CA, USA (fcchou@berkeley.edu); S.-X. Tang is with Department of Electrical Engineering, UC Berkeley, Berkeley, CA, USA and Inria Sophia Antipolis-Méditerranée, France (shuxia_tang@berkeley.edu); X.-Y. Lu is with California PATH, UC Berkeley, Richmond, CA, USA (xiaoyun.lu@berkeley.edu); A. Bayen is with Department of Electrical Engineering, UC Berkeley, Berkeley, CA, USA (bayen@berkeley.edu).

bounded under the impact of preceding human-driven vehicle;

- One challenge of backstepping derivation is "explosion of terms". By induction, complexity of deriving backstepping based controller proposed in this article can be relaxed.

Organization. This article is organized as follows. In section II, the platoon model and problem formulation are detailed. In section III, the proposed controller for automated platooning by backstepping is described, followed by simulations to validate the proposed controller in section IV. Section V concludes the article with our findings and outlook for the future.

II. PROBLEM FORMULATION

We consider a platoon of N automated vehicles, indexed from 1 to N, following a non-automated/human-driven vehicle, indexed 0, see, Fig. 1. For any $i \in \{0, 1, \cdots, N\}$, x_i and L_i denote the position of the rear bumper and the length of the vehicle i, respectively.

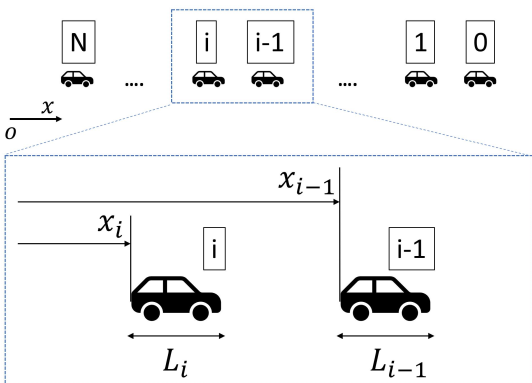

Fig. 1. Vehicle Platooning

Dynamics. The dynamics of each vehicle i, where $i \in \{0, 1, \cdots, N\}$ can be described by the following system of ODEs:

$$\dot{x_i}(t) = v_i(t), \quad (1)$$

$$\dot{v_i}(t) = a_i(t), \quad (2)$$

$$\dot{a}_i(t) = f_i(v_i(t), a_i(t)) + g_i(v_i(t))u_i(t), \quad (3)$$

where

$$f_i(v_i(t), a_i(t)) = -\frac{1}{\tau_i}\left(a_i(t) + \frac{A_{f,i}\rho C_{d,i}v_i(t)^2}{2m_i} + C_{r,i}\right) - \frac{A_{f,i}\rho C_{d,i}v_i(t)a_i(t)}{m_i}, \quad (4)$$

$$g_i(v_i(t)) = \frac{1}{m_i\tau_i}. \quad (5)$$

Here, v_i and a_i are respectively the speed and acceleration of the vehicle i; m_i is the vehicle mass; $A_{f,i}$ is the effective frontal area of the vehicle i; ρ is the air density; $C_{d,i}$ is the aerodynamic drag coefficient; $C_{r,i}$ is the vehicle rolling resistance coefficient; τ_i is the first order response lag time of the powertrain; and u_i denotes the control input actuated on the vehicle i. More details of vehicle longitudinal dynamics can be found in [8].

It is assumed that for every vehicle in the platoon, the state (relative distance from its preceding vehicle, relative speed w.r.t. the preceding vehicle and and its acceleration) is accessible. In practice, relative distance and relative speed can be measured by lidar or radar; speed and acceleration can be measured by odometer and accelerometer. These information can be shared with the other vehicles by wireless communication. The first vehicle in the platoon can only have limited information of the non-automated vehicle ahead using on-board sensors, because the non-automated vehicle ahead is not connected. Hence, only the relative distance and relative speed between the vehicle 1 and the vehicle 0 are used. Other states of vehicle 0, including acceleration ,are not used. Given the fact that the vehicle engine size is limited, the acceleration of the vehicle 0, a_0, is supposed to be bounded.

Assumption 2.1:

$$|a_0(t)| \leq \delta_0. \quad (6)$$

Control objective. The goal is to design the car-following state feedback controllers $u_i, i \in \{0, 1, \cdots, N\}$ so that the controlled automated vehicles in the platoon can follow each preceding vehicle with a constant time-gap, which is the time difference between two consecutive vehicles passing the same place. This kind of gap policy is similar to human's driving behavior that tends to have larger separation to the preceding vehicle when the speed is higher. We define the time-gap error $e_{x,i}$ between consecutive vehicle i and vehicle $i-1$ as follows:

$$e_{x,i}(t) := x_{i-1}(t) - x_i(t) - L_i - hv_i(t), \quad (7)$$

where h is the desired time-gap. It is the distance difference between bumper to bumper distance $(x_{i-1}(t) - x_i(t) - L_i)$ and desired time-gap distance $(hv_i(t))$. We also define the speed error $e_{v,i}$ as follows:

$$e_{v,i}(t) := v_{i-1}(t) - v_i(t). \quad (8)$$

The goal of the controller is to make the time-gap error and speed error converge to zero.

III. AUTOMATED PLATOON CONTROL DESIGN

From the definition of time-gap error $e_{x,i}$, speed error $e_{v,i}$ in (7), (8) and the vehicle dynamics (1)-(3), we have

$$\dot{e}_{x,i}(t) = e_{v,i}(t) - ha_i(t), \quad (9)$$

$$\dot{e}_{v,i}(t) = a_{i-1}(t) - a_i(t), \quad (10)$$

$$\dot{a}_i(t) = f_i(v_i(t), a_i(t)) + g_i(v_i(t))u_i(t). \quad (11)$$

Our goal is to drive vehicles following behind preceding vehicles with a constant time-gap steadily. Therefore, the objective of the car-following controller design is to make $e_{x,i}$ and $e_{v,i}$bounded and eventually converging to zero.

A. Intermediate System

Note that the backstepping approach cannot be directly applied to the dynamics (9)-(11) because the above dynamics is not in the strict feedback form. Therefore, we define

$$z_{i,1}(t) = e_{x,i}(t) - he_{v,i}(t), \tag{12}$$

then the dynamics of $(z_{i,1}(t), e_{v,i}(t), a_i(t))$ is as follows, which is in the strict feedback form:

$$\dot{z}_{i,1}(t) = e_{v,i}(t) - ha_{i-1}(t), \tag{13}$$
$$\dot{e}_{v,i}(t) = -a_i(t) + a_{i-1}(t), \tag{14}$$
$$\dot{a}_i(t) = f_i(v_i(t), a_i(t)) + g_i(v_i(t))u_i(t). \tag{15}$$

In the following section, the controller design for the vehicle 1 in the platoon is firstly described, by backstepping. Then, the backstepping controller for all the other vehicles in the platoon can be designed similarly by induction.

B. Controller design for the vehicle 1

Recall that vehicle 0 is not automated and $a_0(t)$ is bounded. The aim is to make that the vehicle 1 follows the vehicle 0 with a constant time-gap.

1) Stage One: From (13), we have

$$\dot{z}_{1,1}(t) = e_{v,1}(t) - ha_0(t). \tag{16}$$

Let

$$V_{1,1}(t) = \frac{1}{2}z_{1,1}^2(t), \tag{17}$$

which is positive definite. Taking the derivative of $V_{1,1}(t)$, substituting (16) into it and using the virtual control $\bar{e}_{v,1}$ to replace $e_{v,1}$, we obtain

$$\dot{V}_{1,1}(t) = z_{1,1}(t)(\bar{e}_{v,1}(t) - ha_0(t)). \tag{18}$$

By picking $\bar{e}_{v,1}$ as follows:

$$\bar{e}_{v,1}(t) = -\left(k_{1,1} + \frac{h\delta_0}{2\epsilon_{1,1}}\right) z_{1,1}(t), \tag{19}$$

where $k_{1,1}$, $\epsilon_{1,1}$ and following $k_{i,j}$ and $\epsilon_{i,j}$ are all positive design parameters. The second term in the square bracket above is for countering the impact of $a_0(t)$. We plug (19) into (18), then $\dot{V}_{1,1}(t)$ becomes

$$\dot{V}_{1,1}(t) = -k_{1,1}z_{1,1}^2(t) - \frac{h\delta_0}{2\epsilon_{1,1}}z_{1,1}^2(t) - ha_0(t)z_{1,1}(t).$$

Using the fact that $a_0(t)$ is bounded (Assumption 2.1), we obtain following bound on $\dot{V}_{1,1}$

$$\dot{V}_{1,1}(t) \leq -k_{1,1}z_{1,1}^2(t) - \frac{h\delta_0}{2\epsilon_{1,1}}z_{1,1}^2(t) + h\delta_0|z_{1,1}(t)| \tag{20}$$

. By Young's inequality, we know that

$$|z_{1,1}(t)| \leq \frac{z_{1,1}^2(t)}{2\epsilon_{1,1}} + \frac{\epsilon_{1,1}}{2}. \tag{21}$$

We can use the results from Young's inequality to get the bound on $\dot{V}_{1,1}(t)$ as below:

$$\dot{V}_{1,1}(t) \leq -k_{1,1}z_{1,1}^2(t) + h\delta_0\frac{\epsilon_{1,1}}{2}. \tag{22}$$

2) Stage Two: Let

$$z_{1,2}(t) = e_{v,1}(t) - \bar{e}_{v,1}(t). \tag{23}$$

Using (13), (14) and (19), we obtain

$$\dot{z}_{1,1}(t) = z_{1,2}(t) + \bar{e}_{v,1}(t) - ha_0(t), \tag{24}$$
$$\begin{aligned}\dot{z}_{1,2}(t) = &-a_1(t) + a_0(t)\\ &+ \left(k_{1,1} + \frac{h\delta_0}{2\epsilon_{1,1}}\right)(e_{v,1}(t) - ha_0(t)).\end{aligned} \tag{25}$$

We choose the Lyapunov function as

$$V_{1,2}(t) = \frac{1}{2}z_{1,1}^2(t) + \frac{1}{2}z_{1,2}^2(t), \tag{26}$$

which is positive definite. Similarly, by investigating the derivative of $V_{1,2}$, we pick the virtual control $\bar{a}_1$ as

$$\bar{a}_1(t) = z_{1,1}(t) + p_1e_{v,1}(t) + q_1z_{1,2}(t), \tag{27}$$

where

$$p_1 = k_{1,1} + \frac{h\delta_0}{2\epsilon_{1,1}}, \tag{28}$$
$$q_1 = k_{1,2} + \left|1 - k_{1,1}h - \frac{h^2\delta_0}{2\epsilon_{1,1}}\right|\frac{\delta_0}{2\epsilon_{1,2}}, \tag{29}$$

and $k_{1,2}$ is a positive number. From (19) and (27), $\dot{V}_{1,2}(t)$ is bounded as follows:

$$\begin{aligned}\dot{V}_{1,2}(t) \leq &- k_{1,1}z_{1,1}^2(t) - k_{1,2}z_{1,2}^2(t)\\ &+ h\delta_0\frac{\epsilon_{1,1}}{2} + \left|1 - k_{1,1}h - \frac{h^2\delta_0}{2\epsilon_{1,1}}\right|\delta_0\frac{\epsilon_{1,2}}{2},\end{aligned} \tag{30}$$

where Young's inequality is used.

3) Stage Three: Let

$$z_{1,3}(t) = a_1(t) - \bar{a}_1(t). \tag{31}$$

Using (13)-(15), (19) and (27), we obtain

$$\dot{z}_{1,1}(t) = z_{1,2}(t) + \bar{e}_{v,1}(t) - ha_0(t), \tag{32}$$
$$\begin{aligned}\dot{z}_{1,2}(t) = &- z_{1,3}(t) - \bar{a}_1(t) + a_0(t)\\ &+ p_1(e_{v,1}(t) - ha_0(t)),\end{aligned} \tag{33}$$
$$\begin{aligned}\dot{z}_{1,3}(t) = &f_1(v_1(t), a_1(t)) + g_1(v_1(t))u_1(t)\\ &-(1 + p_1q_1)e_{v,1}(t) + (p_1 + q_1)a_1(t)\\ &+ (h + p_1q_1 - p_1 - q_1)a_0(t).\end{aligned} \tag{34}$$

Let the Lyapunov function be

$$V_{1,3} = \sum_{n=1}^{3}\frac{1}{2}z_{1,n}^2(t), \tag{35}$$

which is positive definite. We pick control input u_1 as follows:

$$\begin{aligned}u_1(t) = g_1(v_1(t))^{-1}\Big[&- f_1(v_1(t), a_1(t))\\ &+ p_1z_{1,1}(t) + (2 + p_1q_1)e_{v,1}(t) - (p_1 + q_1)a_1(t)\\ &- k_{1,3}z_{1,3}(t) - |h + p_1q_1 - p_1 - q_1|\frac{\delta_0}{2\epsilon_{1,3}}z_{1,3}(t)\Big],\end{aligned} \tag{36}$$

where $k_{1,3}$ is a positive number, then from Young's inequality, (19), (27) and (36), we obtain

$$\dot{V}_{1,3}(t) \leq \sum_{n=1}^{3} -k_{1,n} z_{1,n}^2(t) + \Gamma_1, \tag{37}$$

where

$$\Gamma_1 = h\delta_0 \frac{\epsilon_{1,1}}{2} + |1-p_1|\,\delta_0 \frac{\epsilon_{1,2}}{2} + |h + p_1 q_1 - p_1 - q_1|\,\delta_0 \frac{\epsilon_{1,3}}{2}. \tag{38}$$

It holds from (35) and (37) that

$$\dot{V}_{1,3}(t) \leq -2\kappa_1 V_{1,3}(t) + \Gamma_1, \tag{39}$$

where

$$\kappa_1 = \min\{k_{1,1}, k_{1,2}, k_{1,3}\}. \tag{40}$$

It can then be derived by the comparison lemma [14] that

$$V_{1,3}(t) \leq \left(V_{1,3}(0) - \frac{\Gamma_1}{2\kappa_1}\right) e^{-2\kappa_1 t} + \frac{\Gamma_1}{2\kappa_1}. \tag{41}$$

This implies that the norm of state errors is ultimately bounded and the convergence rate of the error norm is exponentially fast. By choosing parameters ($k_{1,1}$, $k_{1,2}$, $k_{1,3}$, $\epsilon_{1,1}$, $\epsilon_{1,2}$, $\epsilon_{1,3}$) appropriately, we can make Γ_1 very small, which makes error very close to zero.

4) Closed-loop dynamics of the vehicle 1: Let

$$X_1(t) = [z_{1,1}(t), z_{1,2}(t), z_{1,3}(t)]^T,$$

then by plugging the controller u_1 designed in (36) into the system (32)-(34), we obtain the following closed-loop system dynamics for $X_1(t)$:

$$\dot{X}_1(t) = A_1 X_1(t) + B_1 a_0(t), \tag{42}$$

where $A_1 = \left\{a_1^{(m,n)}\right\}_{3\times 3}$, $B_1 = \left\{b_1^{(m)}\right\}_{3\times 1}$, and

$$\begin{aligned}
&a_1^{(1,1)} = -p_1, a_1^{(1,2)} = 1, a_1^{(1,3)} = 0,\\
&a_1^{(2,1)} = -1, a_1^{(2,2)} = -q_1, a_1^{(2,3)} = -1,\\
&a_1^{(3,1)} = 0, a_1^{(3,2)} = 1,\\
&a_1^{(3,3)} = -k_{1,3} - |h + p_1 q_1 h - p_1 - q_1|\,\frac{\delta_0}{2\epsilon_{1,1}},\\
&b_1^{(1)} = -h, b_1^{(2)} = 1 - p_1 h, b_1^{(3)} = h + p_1 q_1 h - p_1 - q_1.
\end{aligned} \tag{43}$$

Using (31) and (27), the acceleration of vehicle 1 can be rewritten as follows:

$$a_1(t) = K_1^T X_1(t), \tag{44}$$

where

$$K_1 = \left[1 - p_1^2, (p_1 + q_1), 1\right]^T. \tag{45}$$

C. Controller design for the vehicle $i+1$, based on controller design for the vehicle $i, \forall\ 2 \leq i \leq N$

Suppose that the controller is applied to each of the vehicles $j = 1, \cdots, i$ and the closed-loop dynamics of the vehicle i with the designed backstepping controller is:

$$\dot{X}_i(t) = A_i X_i(t) + B_i a_0(t), \tag{46}$$

where $A_i \in \mathbb{R}^{3\times 3}$, $B_i \in \mathbb{R}^{3\times 1}$ and the acceleration a_i is :

$$a_i(t) = K_i^T X_i(t) + \sum_{j=1}^{i-1} M_{i,j} X_j(t), \tag{47}$$

where $K_i \in \mathbb{R}^{3\times 1}$ and $M_{i,j} \in \mathbb{R}^{3\times 1}$. Given i, $M_{i,j}$ is defined as follows:

$$M_{i,j} = \begin{cases} K_i^T & \text{if } j = i \\ M_{i-1,j} - hM_{i-1,j}A_j & \text{if } j \leq i-1 \end{cases}. \tag{48}$$

The corresponding Lyapunov function and its derivative are:

$$V_{i,3}(t) = \sum_{m=1}^{i}\sum_{n=1}^{3} \frac{1}{2} z_{m,n}^2(t), \tag{49}$$

$$\dot{V}_{i,3}(t) \leq \sum_{m=1}^{i}\sum_{n=1}^{3} -k_{m,n} z_{m,n}^2(t) + \Gamma_i. \tag{50}$$

We rewrite the above inequality:

$$\dot{V}_{i,3}(t) \leq -2\kappa_i V_{i,3}(t) + \Gamma_i, \tag{51}$$

where

$$\kappa_i = \min\{k_{1,1}, k_{1,2}, k_{1,3} \cdots k_{i,1}, k_{i,2}, k_{i,3}\},$$

and Γ_i is a function of parameters $k_{1,1}$,$k_{1,2} \cdots k_{i,1}$,$k_{i,2}$,$k_{i,3}$ and $\epsilon_{1,1}$,$\epsilon_{1,2}$,$\epsilon_{1,3}, \cdots, \epsilon_{i,2}$,$\epsilon_{i,3}$. Similarly, by the comparison lemma, we obtain the following bound on $V_{i,3}$:

$$V_{i,3}(t) \leq \left(V_{i,3}(0) - \frac{\Gamma_i}{2\kappa_i}\right) e^{-2\kappa_i t} + \frac{\Gamma_i}{2\kappa_i}. \tag{52}$$

As a result, states until the i-th vehicle can be bounded around the origin. In the following sections, the controller for the $i+1$-th vehicle is designed using backstepping.

1) Stage One: Let

$$V_{i+1,1}(t) = V_{i,3}(t) + \frac{1}{2} z_{i+1,1}^2(t)$$

and pick the virtual control $\bar{e}_{v,i+1}$ as

$$\bar{e}_{v,i+1}(t) = ha_i(t) - k_{i+1,1} z_{i+1,1}(t), \tag{53}$$

where $k_{i+1,1}$ is a positive number, then we have

$$\dot{V}_{i+1,1}(t) = \dot{V}_{i,3}(t) - k_{i+1,1} z_{i+1,1}^2(t). \tag{54}$$

2) Stage Two: We define

$$z_{i+1,2}(t) = e_{v,i+1}(t) - \bar{e}_{v,i+1}(t),$$
$$V_{i+1,2}(t) = V_{i,3}(t) + \frac{1}{2}z_{i+1,1}^2(t) + \frac{1}{2}z_{i+1,2}^2(t),$$

and design the virtual input $\bar{a}_{i+1}$ as follows:

$$\bar{a}_{i+1}(t) =(1 - k_{i+1,1}^2)z_{i+1,1}(t) + (k_{i+1,1} + P_{i+1})z_{i+1,2}(t) + \sum_{j=1}^{i} M_{i+1,j}X_j, \quad (55)$$

where

$$P_{i+1} = k_{i+1,2} + \frac{h\delta_0}{2\epsilon_{i+1,2}}\left|\sum_{j=1}^{i} M_{i,j}B_j\right|. \quad (56)$$

Using Young's inequality, we obtain

$$\dot{V}_{i+1,2}(t) \leq \dot{V}_{i,3}(t) - k_{i+1,1}z_{i+1,1}^2(t) - k_{i+1,2}z_{i+1,2}^2(t) + \frac{\epsilon_{i+1,2}h\delta_0}{2}\left|\sum_{j=1}^{i} M_{i,j}B_j\right|. \quad (57)$$

3) Stage Three: We define

$$z_{i+1,3}(t) = a_{i+1}(t) - \bar{a}_{i+1}(t),$$
$$V_{i+1,3}(t) = V_{i,3}(t) + \frac{1}{2}z_{i+1,1}^2(t) + \frac{1}{2}z_{i+1,2}^2(t) + \frac{1}{2}z_{i+1,3}^2(t).$$

and pick $u_{i+1}(t)$ as follows:

$$\begin{aligned} u_{i+1}(t) = g_{i+1}(v_{i+1}(t))^{-1}\Big\{&-f_{i+1}(v_{i+1}(t), a_{i+1}(t)) \\ &- \left[(2 - k_{i+1,1}^2)k_{i+1,1} + P_{i+1}\right] z_{i+1,1} \\ &+ \left[2 - k_{i+1,1}^2 - (k_{i+1,1} + P_{i+1})P_{i+1}\right] z_{i+1,2} \\ &- (k_{i+1,1} + P_{i+1} + Q_{i+1})z_{i+1,3} + \sum_{j=1}^{i} M_{i+1,j}A_jX_j\Big\}, \end{aligned} \quad (58)$$

where Q_{i+1} is defined as follows:

$$Q_{i+1} =k_{i+1,3} + \left|\sum_{j=1}^{i} M_{i+1,j}B_j - h(k_{i+1,1} + P_{i+1})\sum_{j=1}^{i} M_{i,j}B_j\right| \frac{\delta_0}{2\epsilon_{i+1,3}}. \quad (59)$$

It can then be derived that

$$\dot{V}_{i+1,3}(t) \leq \sum_{m=1}^{i+1}\sum_{n=1}^{3} -k_{m,n}z_{m,n}^2(t) + \Gamma_{i+1}, \quad (60)$$

where

$$\Gamma_{i+1} = \Gamma_i + \left|h\sum_{j=1}^{i} M_{i,j}B_j\right| \frac{\epsilon_{i+1,2}\delta_0}{2} + \left|\sum_{j=1}^{i} M_{i+1,j}B_j - h(k_{i+1,1} + P_{i+1})\left(\sum_{j=1}^{i} M_{i,j}B_j\right)\right| \frac{\epsilon_{i+1,3}\delta_0}{2}. \quad (61)$$

By the comparison lemma, it can be derived that

$$V_{i+1,3}(t) \leq \left(V_{i+1,3}(0) - \frac{\Gamma_{i+1}}{2\kappa_{i+1}}\right)e^{-2\kappa_{i+1}t} + \frac{\Gamma_{i+1}}{2\kappa_{i+1}}.$$

This implies that the norm of state errors is ultimately bounded and the convergence rate of the error norm is exponentially fast. By choosing parameters appropriately, we can make Γ_{i+1} very small, which makes error very close to zero.

4) Closed-loop dynamics of the vehicle $i + 1$: Plugging the controller (58) into the system (32)-(34), we obtain the closed-loop dynamics as follows:

$$\dot{X}_{i+1}(t) = A_{i+1}X_{i+1}(t) + B_{i+1}a_0(t), \quad (62)$$

where $A_{i+1} = \left\{a_{i+1}^{(m,n)}\right\}_{3\times 3}$, $B_{i+1} = \left\{b_{i+1}^{(m)}\right\}_{3\times 1}$, with

$$\begin{aligned} &a_{i+1}^{(1,1)} = -k_{i+1,1}, \quad a_{i+1}^{(1,2)} = 1, \quad a_{i+1}^{(1,3)} = 0, \\ &a_{i+1}^{(2,1)} = -1, \quad a_{i+1}^{(2,2)} = -P_{i+1}, \quad a_{i+1}^{(2,3)} = -1, \\ &a_{i+1}^{(3,1)} = 0, a_{i+1}^{(3,2)} = 1, a_{i+1}^{(3,3)} = -Q_{i+1}, \\ &b_{i+1}^{(1)} = 0, b_{i+1}^{(2)} = -h(K_i^TB_i + \sum_{j=1}^{i-1} M_{i,j}B_j), \\ &b_{i+1}^{(3)} = h(k_{i+1,1} + P_{i+1})\left(\sum_{j=1}^{i} M_{i,j}B_j\right) - \sum_{j=1}^{i} M_{i+1,j}B_j. \end{aligned} \quad (63)$$

And a_{i+1} can be written as follows using (55):

$$a_{i+1}(t) =K_{i+1}^TX_{i+1}(t) + \sum_{j=1}^{i} M_{i+1,j}^TX_j(t), \quad (64)$$

where

$$K_{i+1}^T = [1 - k_{i+1,1}^2, k_{i+1,1} + P_{i+1}, 1]. \quad (65)$$

Note that the closed-loop form and the acceleration of the $i+1$-th vehicle match the formula for i-th vehicle, as shown in (46) and (47). Therefore, by induction, the controller can be designed for every vehicle in the platoon to achieve the time-gap regulation.

IV. Simulation

Two automated vehicles in the platoon, following a human-driven vehicle, are simulated to validate the proposed controller. A speed profile for 120 seconds of the human-driven vehicle is shown as the blue curve in Fig. 2. For the first 80 seconds, the human-driven vehicle accelerates to as high as 21.0 m/s and decelerates to 13.4 m/s at different acceleration/deceleration, simulating speed change in real traffic. It is then followed by a period of constant speed at 20.8 m/s until the end. Two following vehicles, vehicle 1 and vehicle 2 in the platoon, are using the controllers (36) and (58), respectively. The speed responses of vehicle 1 and vehicle 2 are also shown in Fig. 2. Tim-gap errors and speed errors between two consecutive vehicles are shown in Fig. 3 and Fig. 4. In Fig. 3, the blue curve represents the gap error between human-driven vehicle and vehicle 1, and the red curve represents the gap error between vehicle 1 and vehicle 2. In Fig. 4, the blue curve represents the speed error between human-driven vehicle and vehicle 1. The red curve represents the speed error between vehicle 1 and vehicle 2. Results show that the time-gap errors and speed errors can be bounded around zero, even when the human-driven vehicle in the front of automated platoon is changing speed. Time-gap errors and speed errors converge to the origin when the non-automated vehicle is driving at constant speed after 80 seconds, which shows the proposed controller stabilize time-gap car following.

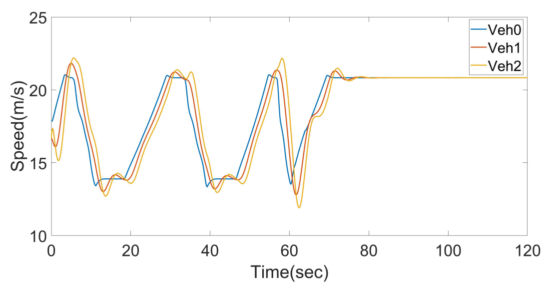

Fig. 2. Speed profiles

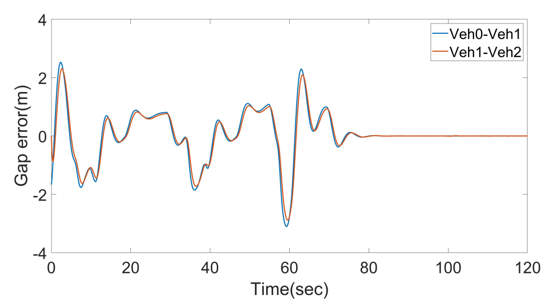

Fig. 3. Gap errors

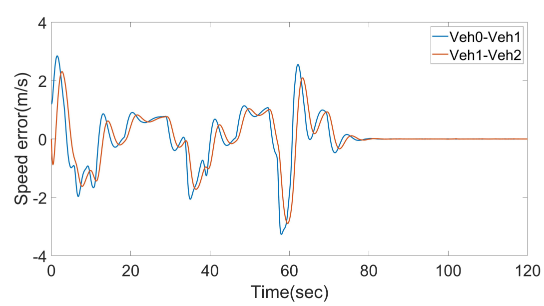

Fig. 4. Speed errors

V. Conclusion and Future Work

Controllers for automated vehicles in a platoon, following a human-driven vehicle, are proposed in this work. A linear coordinate transformation is used to convert the original error dynamics into a strict feedback form, on which the backstepping control method is applied. Acceleration bound of the human-driven vehicle is considered in the controller design. Recursive form of the controller derivation is shown, based on induction, and the performance of the proposed controllers is validated by simulation results.

In the future, the designed controllers will be validated on real vehicles/trucks. Moreover, output feedback controller design based on state observer can also be possible future research topics.

References

[1] S. E. Shladover, "Path at 20history and major milestones," *IEEE Transactions on intelligent transportation systems*, vol. 8, no. 4, pp. 584–592, 2007.

[2] S. E. Li, Y. Zheng, K. Li, and J. Wang, "An overview of vehicular platoon control under the four-component framework," in *2015 IEEE Intelligent Vehicles Symposium (IV)*. IEEE, 2015, pp. 286–291.

[3] S. E. Li, Y. Zheng, K. Li, Y. Wu, J. K. Hedrick, F. Gao, and H. Zhang, "Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities," *IEEE Intelligent Transportation Systems Magazine*, vol. 9, no. 3, pp. 46–58, 2017.

[4] W. Yue, G. Guo, L. Wang, and W. Wang, "Nonlinear platoon control of arduino cars with range-limited sensors," *International Journal of Control*, vol. 88, no. 5, pp. 1037–1050, 2015.

[5] C.-Y. Liang and H. Peng, "Optimal adaptive cruise control with guaranteed string stability," *Vehicle System Dynamics*, vol. 32, no. 4-5, pp. 313–330, 1999.

[6] L. Xiao and F. Gao, "A comprehensive review of the development of adaptive cruise control systems," *Vehicle System Dynamics*, vol. 48, no. 10, pp. 1167–1192, 2010.

[7] V. Milans, S. E. Shladover, J. Spring, C. Nowakowski, H. Kawazoe, and M. Nakamura, "Cooperative adaptive cruise control in real traffic situations," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 1, pp. 296–305, 2014.

[8] R. Rajamani, *Vehicle dynamics and control*. Springer Science & Business Media, 2011.

[9] X.-Y. Lu and J. K. Hedrick, "Heavy-duty vehicle modelling and longitudinal control," *Vehicle System Dynamics*, vol. 43, no. 9, pp. 653–669, 2005.

[10] J. Ploeg, B. T. Scheepers, E. V. Nunen, N. V. de Wouw, and H. Nijmeijer, "Design and experimental evaluation of cooperative adaptive cruise control," in *Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference on*. IEEE, 2011, pp. 260–265.

[11] Y. Wu, S. E. Li, Y. Zheng, and J. K. Hedrick, "Distributed sliding mode control for multi-vehicle systems with positive definite topologies," in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 5213–5219.

[12] Y. Zheng, S. E. Li, K. Li, F. Borrelli, and J. K. Hedrick, "Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 899–910, 2017.

[13] J. Guanetti, Y. Kim, and F. Borrelli, "Control of connected and automated vehicles: State of the art and future challenges," *Annual Reviews in Control*, 2018.

[14] H. K. Khalil, "Noninear systems," *Prentice-Hall, New Jersey*, vol. 2, no. 5, pp. 5–1, 1996.

[15] P. Li, L. Alvarez, and R. Horowitz, "Ahs safe control laws for platoon leaders," *IEEE Transactions on Control Systems Technology*, vol. 5, no. 6, pp. 614–628, 1997.

[16] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems," *IEEE transactions on automatic control*, vol. 45, no. 10, pp. 1893–1899, 2000.